Applications of an Ideal Rainfall-Runoff Chain: Validation and Climate Change

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Abstract: A biased coinflip Ansatz provides a stochastic regional scale land surface climate model of minimum complexity, which represents physical and stochastic properties of an ideal rainfall–runoff chain. The solution yields the empirically derived Schreiber formula as an Arrhenius-type equation of state $W = exp(-D)$. It is associated with two thresholds and combines river runoff *Ro,* precipitation *P* and potential evaporation *N* as flux ratios, which represent water efficiency, $W = Ro/P$, and vegetation states, $D = N/P$. This stochastic rainfall–runoff chain is analyzed utilizing a global climate model (GCM) environment. The following results are obtained for present and future climate settings: (i) The climate mean rainfall-runoff chain is validated in terms of consistency and predictability, which demonstrate the stochastic rainfall–runoff chain to be a viable surrogate model for simulating means and variability of regional climates. (ii) Climate change is analyzed in terms of runoff sensitivity/elasticity and attribution measures.

Key words: Equation of climate states, land surface climates, vegetation, rivers, lakes, stochastic climate models, biased coinflip

1. INTRODUCTION

The rainfall-runoff chain is one of the most relevant sequences of processes linking climate and life on Earth. Therefore, observations and simulations of regional scale climates are analyzed in two ways: first, to describe the underlying processes in terms of fundamental physical and stochastic concepts and to validate simulation models. The Budyko (1974) framework of diagnostics for the catchment scale rainfall-runoff chain appears to be as influential as the Margules-Lorenz energy cycle for the global atmospheric circulation. Measures like non-dimensional water and energy flux ratios have been frequently employed to characterize natural catchments (Sharif et al. 2007; Wang & Takahashi 1998; Zhang et al. 2004 and many more) while regional and global climate model simulations are rarely analyzed with few exceptions (Koster and Suarez 1999, Arora 2002, Fraedrich and Sielmann 2011, Sun et al. 2013). Given the Budyko setting, the aim of this note is threefold, namely to derive an empirically founded climate state equation of the Arrhenius type characterizing an ideal rainfall-runoff chain (Schreiber 1904, Fraedrich 2010), to validate its first (and second) moments in a global climate model (GCM) environment (Fraedrich and Sielmann 2011) and, finally, to analyze global change and estimate its attribution. Section 2 introduces basics of the ideal rainfall-runoff chain, which is followed by a validation analysis (section 3) based on measures of regional scale consistency and predictability using state of the art global climate model (GCM) simulations. Section 4 presents climate change analyzed in terms of sensitivity, elasticity and attribution measures.

2. THE BUDYKO-SCHREIBER FRAMEWORK

Budyko's (1974) framework of climate analysis characterizes the Earth's surface cover by climate state variables, which are defined in terms of three basic non-dimensional ratios comprising water and energy fluxes separately and in combination thereby satisfying the respective long-term mean balance equations.

Water-energy balance: Water is supplied by precipitation *P* and balanced by streamflow or runoff *Ro* and evapotranspiration *E*. Energy is supplied by net radiation *N* and balanced by sensible and latent heat fluxes, *H* and *E* (energy flux units are in water flux equivalents):

$$P = Ro + E \tag{1}$$
$$N = H + E \tag{2}$$

Flux partitioning into runoff Ro plus evaporation E, and into fluxes of sensible heat H plus moisture E, is due to processes comprising the rainfall–runoff chain linking atmosphere, vegetation and soil. To understand these processes they are interpreted by a theory of an ideal rainfall-runoff chain revealing an equation of state for land surface climates.

Ideal rainfall-runoff chain: The climate variables are functionally related by an equation of state for ideal land surface climates that, not unlike the ideal gas law, has been discovered (Schreiber 1904). A derivation (Fraedrich 2010) leads to an Arrhenius-type equation of state that, representing physical *and* stochastic properties of the rainfall–runoff chain, provides a regional scale model of minimum complexity. Schreiber's (1904) aridity-runoff relation represents an equilibrium solution of the rainfall-runoff chain. The chain commences with a fast stochastic water reservoir of small capacity representing interception in short time intervals. It feeds a slow (almost stationary) soil moisture reservoir of large capacity balancing its runoff Ro after long-term averaging. Parameterizing the fast reservoir's capacity by (the water equivalent of) net radiation N available for evaporation leads to a biased coin-flip surrogate for its 'full' or 'empty' states, when rainfall is 'larger' or 'smaller' than the capacity. Rainfall surplus from the fast reservoir's 'full' state feeds the slow reservoir; the residual evaporates so that the fast reservoir can start as 'empty'. Rainfall below capacity evaporates completely and leaves the energy surplus as sensible heat H; now the fast reservoir can start again as 'empty'. Using the biased coinflip occurrence probabilities from a maximum entropy (exponential) distribution of precipitation yields Schreiber's (1904) empirical formula, which provides a water-energy flux balance closure of Arrhenius-type. As an equation of state,

$$W = exp(-N/P) \tag{3}$$

it relates the runoff-rainfall ratio $W = Ro/P$ to the occurrence probability of water supply P exceeding energy supply N (net radiation or water demand). In this sense an increasing runoff-rainfall ratio W is linked to a decreasing dryness $D = N/P$ (increasing wetness $1/D$).

(i) *Dryness ratio*: Following Budyko (1974) the dryness ratio characterizes the geobotanic states of the climate, which combines energy and water fluxes by relating water demand (energy supply) to water supply,

$$D = N/P.$$

This ratio is a quantitative geobotanic measure of the climate-vegetation relation (Budyko 1974): Tundra, $D < 1/3$, and forests, $1/3 < D < 1$, are energy limited because available energy N is low, so that runoff exceeds evaporation for given precipitation, $E \sim N$. Steppe and Savanna, $1 < D < 2.0$, semi-desert $2.0 < D < 3.0$, and desert $3.0 < D$, are water limited climates, where the available energy is so high that water supplied by precipitation evaporates, which then exceeds runoff, $E \sim P$.

(ii) *Runoff–rainfall ratio* (excess water): Relating water fluxes to their supply by rainfall yields the runoff ratio

$$W = Ro / P.$$

It describes streamflow of rivers on a regional or catchment scale. It characterizes the natural rainfall-runoff chain in terms of relative excess rainfall $(P – E) / P$, which represents the fraction of water unused by an ecosystem (but available for terrain formation, Milne et al. 2002) compared to the total water supply. In this sense the runoff-rainfall ratio is also a measure of the water efficiency

of an ecosystem. Note that negative excess rainfall occurs, if evaporation is supported by hydrologic and not by atmospheric processes.
(iii) *Energy flux ratio* (excess energy): Relating energy fluxes to their supply yields a relative energy flux ratio

$$U = H / N,$$

which describes the fraction of energy unused by the system when compared to its supply. That is, the relative excess energy $(N – E) / N$ (available for photosynthesis, Milne et al. 2002) which, not unlike the water efficiency W, is a measure of the energy efficiency of an ecosystem. Both water and energy excess (or efficiency) describe proportions of available water and energy which, remaining unused, appear to be relevant to identify the causes of climate and basin change (see section 4). Note also, that the net radiation as the energy supply is also interpreted (Budyko 1974) as a water demand or potential evapotranspiration. In summarizing (Fig. 1), the ideal rainfall-runoff chain is characterized by functional relations between ratios of long term mean fluxes: excess water, $W = exp(-D)$, or energy, $U = 1 – \{1 – exp(-D)\} / D$, depending on the dryness ratio D. Their combination in (W,U)-plane, $U = 1 + (1 – W) / ln(W)$, is used to attribute change $(dU / dW) = – \{ exp(D) – (D +1) \} / D^2$. For future reference: a superscript ()* is used to refer to the ideal rainfall-runoff chain (eq. 3.)

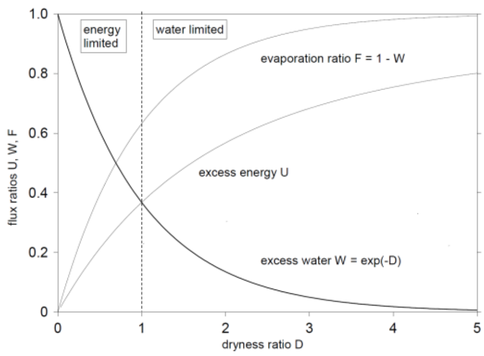

Figure 1. Idealized long term mean rainfall-runoff diagram (Schreiber 1904): Functional relation of excess water W (runoff-rainfall ratio) and excess energy U depending on dryness ratio D; evaporation ratio F is also included.

3. CLIMATE MEANS AND RUNOFF: VALIDATION OF FIRST MOMENTS

Observed and simulated climates are subjected to Budyko’s (1974) framework of analysis for validation and for comparison with the equation of state for ideal land surface climates (eq. 3). The climates associated with the stochastic land surface model of rainfall–runoff chain are analyzed using simulations of a coupled atmosphere-ocean global climate model (GCM). GCM simulations are based on a state of the art model providing long term nine-neighbor means of continental grid points simulated by a 20th century control run (1958–2001, IPCC AR4 MPI-ECHAM5-T63L31 coupled to MPI-OMGR1.5L40 GR1, run on a NEC-SX with resolution T63 (1.875°), N48) using observed anthropogenic forcings by CO-2, CH-4, N-2O, CFCs, O-3 and sulfate (Roeckner et al. 2006; Hagemann et al. 2006). The annual mean data sets form the basis of the subsequent analyses following Fraedrich and Sielmann (2011).

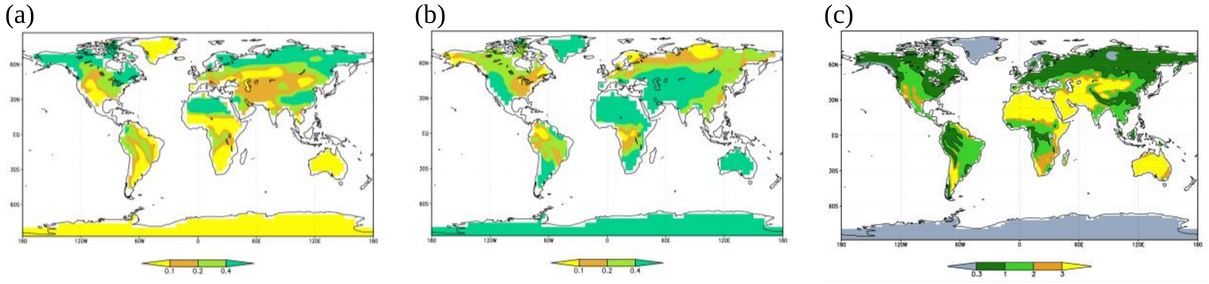

Figure 2. Simulated long term mean rainfall-runoff chain (Global Climate Model ECHAM5 20C): Geographical distributions of (a) excess water W (runoff-rainfall ratio), (b) excess energy U, and (c) dryness ratio D.

Koeppen's climate classes: The global distribution of Budyko's dryness ratio *D*, which is the fundamental climate state parameter of the rainfall–runoff chain, is shown in Fig. 2 (Fraedrich and Sielmann 2011) . For comparison, the commonly used Koeppen classification (Koeppen 1936) is also presented (Fig. 3) with its main types of tropical, dry, subtropical, temperate, boreal (cold) and ice climates (see, for example, Fraedrich et al. 2001), which are commonly attached with the letters A to F, respectively. Note that the dryness ratio *D* is a continuously varying parameter while the Koeppen classes are discrete. The main difference between both classifications occurs in the tropical and temperate climates related to forest vegetation; these are clearly distinct climate types in Koeppen while, for Budyko's dryness ratio, forests (ranging from tropical rain to temperate vegetation) occur within a relatively small *D*-interval.

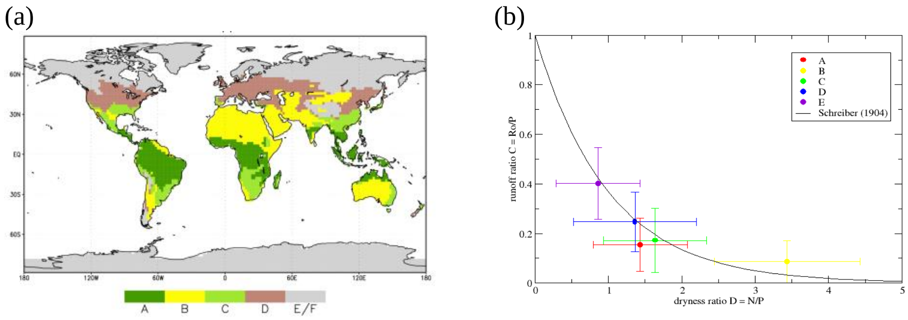

Figure 3. Present day surface climates simulated: (a) Koeppen (1936) climates and (b) their locations in the idealized rainfall-runoff diagram aligning Schreiber's (1904) formula (Fig. 1).

Validation: The ideal rainfall–runoff chain is validated in two steps: (i) Consistency is assessed comparing the ideal rainfall-runoff chain with the coupled GCM simulation by sampling the simulated Koeppen classes (A to F) in bins of the simulated dryness ratio *D*. The sample averages and standard deviations (vertical and horizontal axes centered on the means) show (Fig. 3) that the discrete Koeppen climate types are well aligned along the ideal rainfall–runoff chain's dryness dependent runoff ratio (eq. 3, dashed). The geobotanic dryness *D* and Koeppen classes may fit Schreiber's equation better after suitably regrouping the Koeppen climate classes (including the subclasses; see Hanasaki et al. 2008).

(ii) Predictability is analyzed comparing the simulated runoff *Ro* with the runoff *Ro** derived by Schreiber's equation of state using simulated dryness ratio *D* and precipitation *P* (Fig. 4). Predictability is verified as demonstrated by the (*Ro*, *Ro**)-scatter plot. The consistency between GCM-simulated dryness *D* dependent runoff-rainfall ratios *W* is also presented sampled in *W*-bins with means and standard deviations (large dots and horizontal lines) being compared with Schreiber's formula (eq. 3, dashed) in (*W*, *D*)-space.

In summarizing, the stochastic rainfall–runoff chain's consistency with the phenomenological (Koeppen) climate classes and its predictability (within a consistent data set) for runoff , excess rainfall or energy encourages further exploration for analyzing land surface climates.

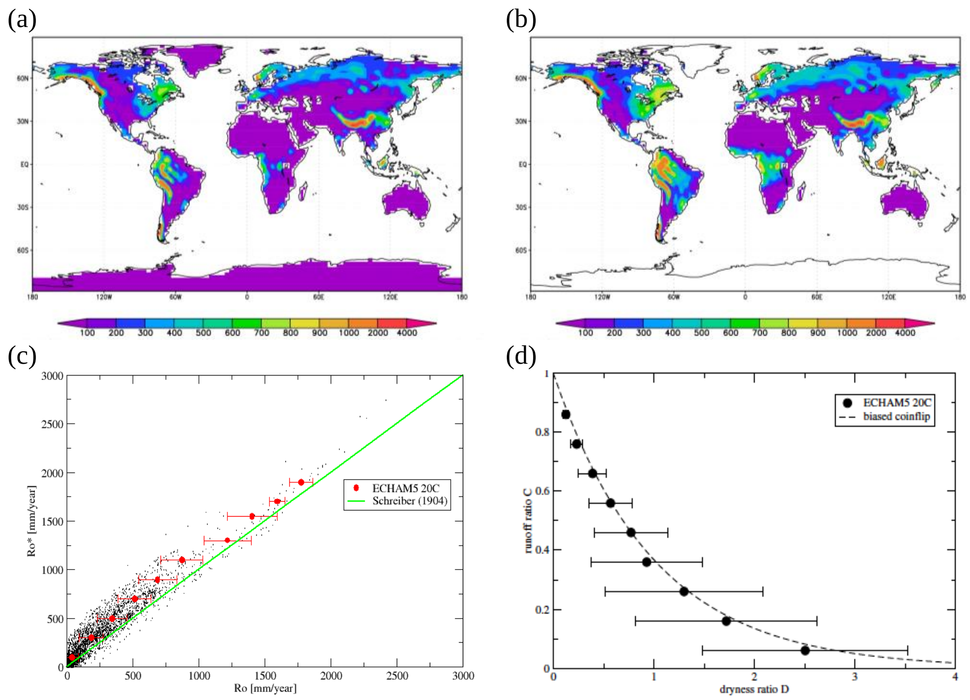

Figure 4. Validation of the rainfall–runoff chain: The geographical distributions of mean streamflow (a) Ro simulated (ECHAM5 20C), and (b) Ro derived by Schreiber's formula using simulated dryness D and rainfall P, are (c) compared in a (Ro, Ro*)-scatter plot. (d) GCM-simulated dryness D and excess rainfall W sampled in W-bins (means, standard deviations as dots, horizontal lines) align Schreiber's formula (dashed).*

4. CLIMATE CHANGE ANALYSIS

Budyko's (1974) framework of analysis is now employed to analyze climate change simulated by state of the art GCM simulations for present day (section 3) and double CO-2 conditions prescribed by A1B–scenario (see Roeckner et al. 2006). Sensitivity and elasticity measures are deduced and interpreted in terms of the ideal rainfall-runoff chain (eq. 3) before quantifying the change attribution.

Sensitivity and elasticity: Sensitivity to small changes of boundary conditions is deduced, because it is of great relevance for interpreting climate change estimates and model performance. That is, runoff changes are described to first order

$$\Delta Ro = Ro_P \Delta P + Ro_N \Delta N \qquad (4)$$

by partial differentials, Ro_P and Ro_N. For example, the Schreiber-Budyko Ansatz as an ideal or reference rainfall-runoff chain, yields $Ro_P = (1+D)\exp(-D)$ and $Ro_N = -\exp(-D)$. Rearrangement shows that the runoff sensitivity, $\Delta Ro/Ro$, depending on the sensitivities of water supply, $\Delta P/P$, and demand, $\Delta N/N$:

$$\Delta Ro/Ro = \varepsilon(Ro, P)\Delta P/P + \varepsilon(Ro, N)\Delta N/N \qquad (5)$$

are weighted by the system's runoff elasticities, ε. These, in turn, are also related to water supply and demand (see Dooge 1992) and add up to one, $\varepsilon(Ro, P) + \varepsilon(Ro, N) = 1$. Now, simulated elasticities for changing climates are interpreted by the ideal rainfall-runoff chain.

Example: For fixed water demand (or energy supply), N = constant, a model output under a climate change scenario with changing precipitation only, ΔP (e.g. a shift of the monsoonal circulation), allows estimates of rainfall dependent runoff elasticity by a model simulation and the ideal rainfall-runoff chain:
Simulated/observed runoff elasticity is determined by runoff change ΔRo induced by rainfall change ΔP given by the differences of climate mean runoff Ro and precipitation P between scenario and control (or present and past) data sets:

$$\varepsilon(Ro, P) = \Delta Ro/Ro \,/\, \Delta P/P.$$

Ideal runoff elasticity, which depends only on the dryness and runoff ratio relation $D^* = -ln(W)$, is taken from the rainfall efficiency $W = Ro/P$ simulated by the control experiment:

$$\varepsilon^*(Ro, P) = 1 + D.$$

Consequences are as follows: (i) Small differences between two elasticity estimates, ε and ε^*, indicate that the simulated response to climate change is well represented by the processes described by the ideal rainfall-runoff chain. (ii) Net radiation dependent runoff elasticity can also be deduced, because $\varepsilon^*(Ro, N) = -D$ satisfies the balance $\varepsilon^*(Ro, P) + \varepsilon^*(Ro, N) = 1$, if N and P are uncorrelated. Fig. 5 demonstrates this in the global change GCM environment.

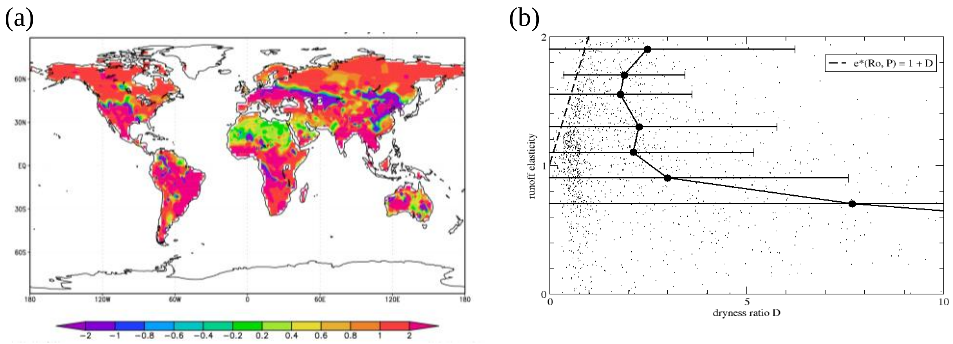

Figure 5. Rainfall induced runoff elasticity $\varepsilon(Ro,P)$ simulated by ECHAM5 20C: (a) Geographical and (b) dryness D dependent distributions (scatter plot); runoff elasticitiy ε^* based on Schreiber's formula (dashed).

Attribution: Further insight into the runoff-rainfall processes underlying climate and basin change is gained by employing the Budyko framework (following Tomer and Schilling 2009, see also Renner et al. 2012). Utilizing relative excess rainfall and energy change, dW and dU, provides estimates the change attribution.
(i) *Climate* change is due to changing precipitation P or net radiation N (linked by dryness ratio $D = N/P$) and leads to shifts in water and energy excess (W,U) in opposite directions in (W,U)-space. The idealized rainfall-runoff chain shows a decreased (increased) dryness D associated with an increased (decreased) relative excess water W^*, while excess energy has decreased (increased) along the negative diagonal. This is, because $W^* = exp(-D)$ and $U^* = 1 - \{1 - exp(-D)\} / D$, and thus $(dU/dW)^* \sim -1$.
(ii) *Basin* change is characterized by changes in evapotranspiration from vegetation or soil. It introduces shifts in excess water W and energy U towards increased W and U, or decreased W and U, respectively. A change of the evaporation ratio, $F = E/P$, shifts excess rainfall, $W = 1 - F$, and excess energy, $U = 1 - F / D$, in the same directions along the main diagonal; that is $dU/dW \sim 1$.
(iii) *Attribution* of climate or basin change is measured by the ratio of excess energy dU and excess rainfall change dW which, in the ideal case, is given by the attribution or slope $(dU / dW)^* = -\{ exp(D) - (D + 1) \} / D^2 < 0$ (see Fig. 6).

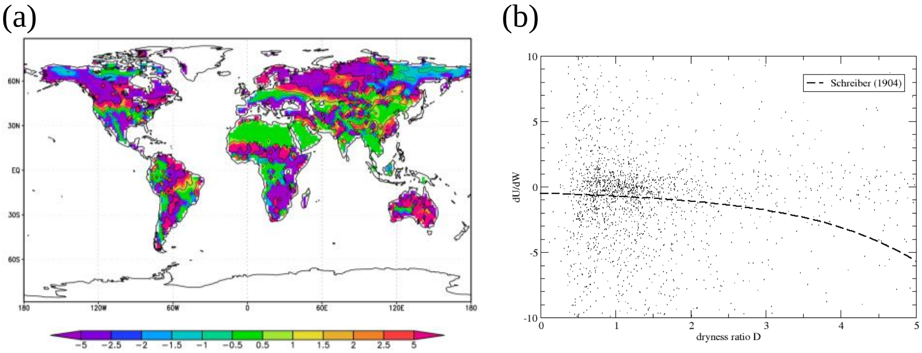

Figure 6. Attribution of climate change (ECHAM5 A1B-scenario) based on the ratio dU/dW of changes of relative excess energy dU and rainfall dW: (a) Geographical and (b) dryness D dependent distributions (scatter plot); runoff-rainfall attribution based on Schreiber's formula (dashed).

5. CONCLUSION AND OUTLOOK

We have characterized the Earth's surface cover by climate state parameters, employing Budyko's (1974) framework of climate analysis in terms of non-dimensional water and energy flux ratios and using an equation of state for ideal land surface climates (Schreiber 1904). Validation of Schreiber's equation, which has been derived as an ideal rainfall-runoff chain employing a stochastic biased coinflip model, is performed within a coupled Global Climate Model (GCM) environment (supplementing a previous regional model analysis, Sun et al. 2013), because state of the art models provide physically consistent datasets. Validation analyses show that, on catchment scale, the stochastic model performs as an ideal rainfall-runoff chain (Schreiber's equation); it may serve as a suitable concept for analyzing the Earth's near surface climate, which comprises vegetation, rivers (and lakes) as its main natural features and includes variability and change.

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REFERENCES

Arora, V. 2002. The use of the aridity index to assess climate change effect on annual runoff. J. of Hydrology, **265**, 164–177.

Budyko, M.I. 1974. Climate and Life, Academic Press, 508pp.

Dooge, J.C.I. 1992. Sensitivity of runoff to climate change: A Hortonian approach. Bull. Am. Metorol. Soc., **73**, 2013–2024.

Fraedrich, K., Gerstengarbe, F. W. and Werner, P. C. 2001. Climate shifts in the last century. Clim. Change 50, 405–417.

Fraedrich, K. 2010. A stochastic parsimonious water reservoir: Schreiber's 1904 equation. J. Hydromet., **11**, 575 – 578.

Fraedrich, K. and F. Sielmann 2011. An equation of state for land surface climates. International Journal of Bifurcation and Chaos 21, 3577–3587.

Fraedrich, K. 2013. A minimalist model of terminal lakes: Qinghai Lake (China) and Lake Chad (N-Africa). Hydrology Research, in revision.

Hagemann, S., K. Arpe, K. and E. Roeckner, 2006. Evaluation of the hydrological cycle in the ECHAM5 model. J. Clim. 19, 3810–3827.

Hanasaki, N., Kanae, S., Oke, T., Masuda, K., Motoya, K., Shirakawa, N., Shen, Y. and Tanaka, K. 2008. An integrated model for the assessment of global water resources - Part 1: Model description and input meteorological forcing. Hydrol. Earth Syst. Sci. 12, 1007–1025.

Koeppen, W. 1936. Das geographische System der Klimate – Handbuch der Klimatologie, Vol. 1, Part C, Gebr. Borntraeger Verl., Berlin, p. 388.

Koster, R.D. and M. J. Suarez 1999. A simple framework examining the interannual variability of land surface moisture fluxes. J. Climate 12, 1911–1917.

Milne, B., V. Gupta, V., and C. Restrepo 2002. A scale invariant coupling of plants, water, energy, and terrain. Ecoscience, 9, 191–199.

Renner, M., R. Seppelt, and C. Bernhofer 2012. Evaluation of water-energy balance frameworks to predict the sensitivity of streamflow to climate change. Hydrol. Earth Syst. Sci., 16, 1419–1433.

Roeckner, E., M. Lautenschlager, and H. Schneider 2006. "IPCC-AR4MPI-ECHAM5-T63L31MPI-OMGR1.5L40 SRESA1B run no. 1: Atmosphere monthly mean values MPImet/MaD Germany," World Data Center for Climate [doi:10.1594/WDCC/EH5-T63L31-OM-GR1.5L40-A1B-1-MM].

Schreiber, P. 1904. Über die Beziehungen zwischen dem Niederschlag und der Wasserführung der Flüsse in Mitteleuropa. Meteorolog. Z. 21, 441–452.

Sharif, H. O., W. Crow, N. L. Miller, E. F. Wood 2007. Multidecadal high-resolution modeling of the Arkansas-Red River basin. J. Hydromet. 8, 1111–1127.

Sun R., X. Zhang, Y. Sun, Du Zheng, K. Fraedrich 2013. SWAT-based streamflow estimation and its responses to climate change in Kadongjia River Watershed, South Tibet, China. J. Hydromet., in revision.

Tomer M.D. and K. E. Schilling 2009. A simple approach to distinguish land-use and climate-change effects on watershed hydrology. J. Hydrol. 376, 24–33.

Wang, Q. and H. Takahashi 1998. A land surface water deficit model for an arid and semiarid region: Impact of desertification on the water deficit status in the Loess Plateau, China. J. Climate 12, 244–257.

Yang, D., F. Sun, Z. Liu, Z. Cong, and Z. Lei 2006. Interpreting the complementary relationship in non- humid environments based on the Budyko and Penman hypotheses. Geophys. Res. Lett., 33, L18402, doi:10.1029/ 2006GL027657.

Zhang, L., K. Hickel, W. R. Dawes, F.H.S. Chiew A. W. Western, and P. R. Briggs 2004. A rational function approach for estimating mean annual evapotranspiration. Water Resources Research, 40, W02502, doi:10.1029/2003WR002710.