The atmospheric circulation and states of maximum entropy production

Axel Kleidon
Department of Geography, University of Maryland, College Park, Maryland, USA

Klaus Fraedrich, Torben Kunz, and Frank Lunkeit
Meteorologisches Institut, Universität Hamburg, Hamburg, Germany

Received 7 August 2003; revised 8 October 2003; accepted 7 November 2003; published 12 December 2003.

[1] Energy balance models suggest that the atmospheric circulation operates close to a state of maximum entropy production. Here we support this hypothesis with sensitivity simulations of an atmospheric general circulation model. A state of maximum entropy production is obtained by (i) adjusting boundary layer turbulence and (ii) using a sufficiently high model resolution which allows sufficient degrees of freedom for the atmospheric flow. The state of maximum entropy production is associated with the largest conversion of available potential energy into kinetic energy which is subsequently dissipated by boundary layer turbulence. It exhibits the largest eddy activity in the mid latitudes, resulting in the most effective transport of heat towards the poles and the least equator-pole temperature difference. These results suggest that GCMs have a fundamental tendency to underestimate the magnitude of atmospheric heat transport and, therefore, overestimate the equator-pole temperature gradient for the present-day climate, for the response to global climatic change, and for atmospheres of other planetary bodies. *INDEX TERMS:* 3307 Meteorology and Atmospheric Dynamics: Boundary layer processes; 3319 Meteorology and Atmospheric Dynamics: General circulation; 3337 Meteorology and Atmospheric Dynamics: Numerical modeling and data assimilation; 3379 Meteorology and Atmospheric Dynamics: Turbulence. **Citation:** Kleidon A., K. Fraedrich, T. Kunz, and F. Lunkeit, The atmospheric circulation and states of maximum entropy production, *Geophys. Res. Lett.*, *30*(23), 2223, doi:10.1029/2003GL018363, 2003.

1. Introduction

[2] The Earth's atmosphere is a thermodynamic heat engine which transports heat from the tropics to the poles. The differential heating of the surface, with tropical regions receiving more solar radiation than polar regions, causes a horizontal temperature gradient to develop, which generates available potential energy (APE) which can subsequently be converted into kinetic energy (KE, i.e., atmospheric motion) and is eventually extracted by boundary layer turbulence due to surface friction. Several studies have suggested that the atmospheric circulation works close to its maximum intensity, that is, at a state where the rates of energy conversions are at a maximum, corresponding to a state of maximum entropy production (*MaxEP*, or *MEP*) [*Lorenz*, 1960; *Paltridge*, 1975, 1978; *Grassl*, 1981; *Lorenz et al.*, 2001; *Ozawa et al.*, 2003]. The evidence of a maximum in entropy production can easily be demonstrated by considering the two extremes of atmospheric poleward heat transport. With no heat transport, the equator-pole temperature gradient is at its largest, but since no heat is transported, no entropy is produced by poleward heat transport. At the other extreme of very large heat transport, the temperature difference becomes very small, so that the conversion of heat occurs at roughly the same temperatures and little entropy is produced. Consequently, there should be a maximum in entropy production for an intermediate value of heat transport, resulting in an intermediate temperature gradient between the tropics and the poles. What *Lorenz* [1960], *Paltridge* [1975, 1978] and others have shown with energy balance models is that the intensity of the atmospheric circulation and the associated equator-pole temperature gradient predicted by employing *MEP* are close to observations. On the same line, intriguing results from *Lorenz et al.* [2001] suggest that *MEP* can also reproduce the equator-pole temperature gradients for the atmospheres of Mars and Titan. However, the *MEP* hypothesis is not widely accepted, partly due to a lack of theoretical foundation and partly due to the seeming contradiction of a simple, emergent outcome from the complex details of the atmospheric circulation.

[3] A theoretical justification for *MEP* has recently been derived from information theory [*Dewar*, 2003; see also *Lorenz*, 2003]. *Dewar* [2003] applied *Jaynes'* [1957] information theory formalism of statistical mechanics to non-equilibrium systems in steady state to show that out of all possible macroscopic stationary states compatible with the imposed constraints (e.g., external forcing, local conservation of mass and energy, global steady-state mass and energy balance), the state of maximum entropy production is selected because it is statistically the most probable, i.e., it is characteristic of the overwhelming majority of microscopic paths allowed by the constraints. What Dewar's work implies is that *MEP* should emerge from simulation models of non-equilibrium systems, such as the atmospheric circulation, by two different means: For a macroscopic parameterization of a dynamic process, a *MEP* state can be obtained by optimizing a parameter which is used instead of a detailed representation of the degrees of freedom of the system dynamics (we refer to this as a type I parameterization). One example is the simple parameterization of heat transport used in energy balance models as discussed above. If the microscopic dynamics of the system are explicitly

0094-8276/03/2003GL018363$05.00

simulated, *MEP* should emerge from the model simulation if sufficient degrees of freedom are represented in the model (we refer to this as type II parameterization).

[4] Here we test the *MEP* hypothesis and how it emerges in models by the two different types of parameterizations with an atmospheric General Circulation Model (GCM). In a GCM, the spatial degrees of freedom for the atmospheric flow [e.g., *Fraedrich et al.*, 1995] associated with large-scale eddies in the mid latitudes are explicitly allowed for by the primitive equations of fluid dynamics, but the spatial structures of turbulence are constrained by the model's resolution. We test the emergence of *MEP* (type II) associated with horizontal turbulence by varying the spatial resolution of the model. An increased resolution provides a wider range of atmospheric modes, or degrees of freedom, so that the atmospheric circulation can adjust to MEP states. One would, therefore, expect entropy production to increase with the spectral resolution of the model until sufficient degrees of freedom are represented. On the other hand, the dissipation of kinetic energy into heat associated with boundary layer turbulence is usually represented by a comparably simple parameterization in GCMs since it generally occurs at a much smaller spatial scale. For this macroscopic parameterization a state of *MEP* should result by optimization of a parameter that characterizes the degrees of freedom.

2. Methods

[5] We use the PUMA-1 atmospheric General Circulation Model [*Fraedrich et al.*, 1998]. The PUMA-1 GCM is a simple, multi-level spectral GCM [*Hoskins and Simmons*, 1975; *James and Gray*, 1986; *James and James*, 1989; *Held and Suarez*, 1994]. It consists of the dynamical core of a GCM only, that is, there are no explicit calculations of the radiation or water balance. The dynamics of the atmosphere are forced by diabatic heating/cooling and by boundary layer friction, which appear as linear terms in the thermodynamic and momentum equations. The diabatic heating drives the atmospheric circulation by relaxing its temperature towards a radiative-convective equilibrium state on a time scale of τ_{HEAT} = 30 days for the upper layers, τ_{HEAT} = 10 days for the second-lowest model layer and τ_{HEAT} = 5 days for the lowest model layer. The effects of boundary layer turbulence and friction are modelled by Rayleigh friction in the lowest model layer, that is, $(u, v)/\tau_{FRIC}$ describes the decelleration of the air in the lowest model layer, with (u, v) being the horizontal wind speed components. In its control setup, the model uses a value of τ_{FRIC} = 1 day in T21 spectral resolution and five vertical layers of equal mass.

[6] We modified the model by adding diagnostic entropy flux calculations by computing the change in heat content of each grid box on each level (i.e. the net energy flux) divided by the respective temperature at which the fluxes are exchanged:

$$F_\sigma = 1/T\, \partial/\partial t\left(c_p\, \rho\, T\right) \tag{1}$$

The fluxes computed by equation (1) include the different contributions by radiative heating, subgrid-scale diffusion, and mixing. The time-averaged global integral of all entropy fluxes then yields the steady-state rate of entropy produced by the atmospheric circulation:

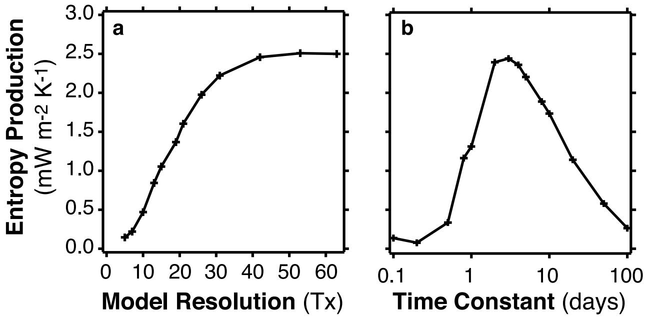

Figure 1. Entropy production by atmospheric heat transport as a function of (a) the spatial resolution, expressed by the model's spectral triangular truncation number; and (b) the intensity of boundary layer turbulence, determined by a friction time constant, with larger values representing less friction. For (a), the simulations at each resolution were used with the value of τ_{FRIC} for which entropy production was at a maximum. For (b), the simulations conducted at T42 resolution were used.

$$\sigma = \int\!\!\int F_\sigma\, dV\, dt \tag{2}$$

An alternative way to calculate entropy production is from the difference in entropy fluxes associated with the diabatic heating and cooling terms in the thermodynamic equation. We found in our simulations that both ways lead to the same values of global entropy production.

[7] We calculated entropy production for a range of simulations of different resolutions and different values of τ_{FRIC}. For each of the resolutions used (spectral resolutions of T5, T7, T10, T13, T15, T19, T21, T26, T31, T36, T42, T53, and T63), we used values for τ_{FRIC} of 0.1, 0.2, 0.5, 0.8, 1, 2, 3, 5, 8, 10, 20, 50, and 100 days. Note that for the latter case it would not be surface friction *per se* which would adjust to a state of *MEP*, but rather the associated characteristics of boundary layer turbulence. However, the model used here does not distinguish between these two aspects. Each simulation was run for 10 years, with the first 2 years discarded to avoid spin-up effects. We use a setup representative of Northern hemisphere summer conditions.

3. Results

[8] As hypothesized in the introduction, entropy production increases with higher model resolution (type II, Figure 1a) and saturates at a resolution of T42. For boundary layer turbulence, a state of *MEP* is achieved through optimization of the Rayleigh friction time scale τ_{FRIC} (type I, see Figure 1b).

[9] The maximum in entropy production associated with τ_{FRIC} can be understood from simple, dynamical considerations. At very high friction, kinetic energy is rapidly extracted which prevents mid-latitude eddies to grow and results in little motion near the surface. Therefore, little mixing of air masses occurs in the mid latitudes which means that APE is not effectively converted into KE and

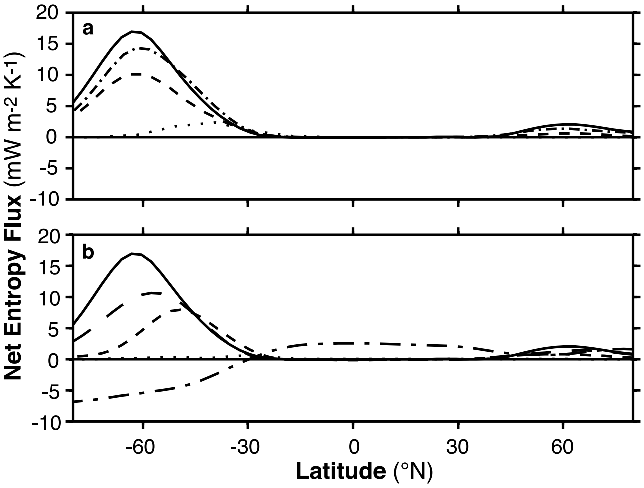

Figure 2. Latitudinal variation of net entropy fluxes for different model resolutions and boundary layer turbulence. (a) Model simulations using different resolution with optimum values for τ_{FRIC}, for T10 resolution (dotted), T21 resolution (dashed), T31 resolution (dash-dotted) and T42 resolution (solid). (b) Model simulations using different intensities of boundary layer turbulence at T42 resolution for the following values of τ_{FRIC}: 0.1 days (dotted), 1 day (short dashes), 3 days (solid), 10 days (long dashes), and 100 days (dash-dotted). The simulations are conducted under prescribed northern hemisphere summer conditions.

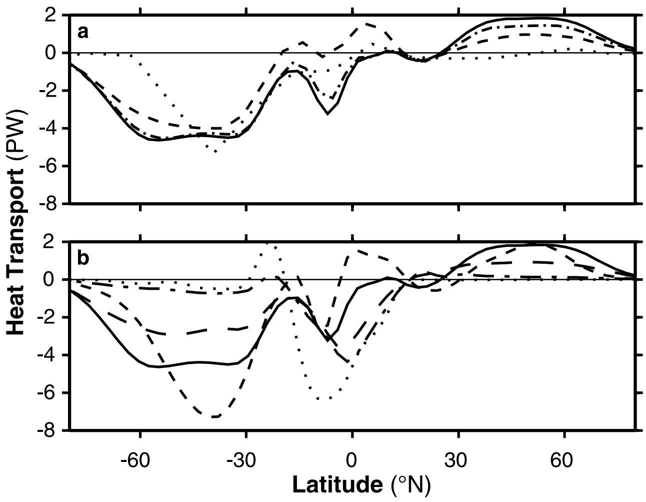

Figure 3. Latitudinal variation of poleward atmospheric heat transport for different model resolutions and boundary layer turbulence, with notations as in Figure 2. Negative values of heat transport correspond to southward transport of heat.

consequently, entropy production is low. The low friction extreme can be explained by the 'barotropic governor' [*James and Gray*, 1986]. Lower friction leads to a more barotropic mean zonal flow which is more stable to baroclinic conversions. The increased kinetic energy of the zonal flow then feeds back to the conversion of available energy to baroclinic energy release, leading to overall decreased eddy activity. This state is also characterized by low mixing of air masses and low entropy production. Therefore, between these two extremes, there is an intermediate value of τ_{FRIC} for which entropy production, baroclinic activity, and the associated conversions of APE to KE to heat are at a maximum. For both types of parameterizations the *MEP* state is associated with strongest baroclinic activity which also shifted furthest towards the poles (Figure 2).

[10] The different intensitites of baroclinic activity associated with different states of entropy production lead to important consequences for the simulated climates. With increasing values of entropy production, heat is more effectively transported polewards (Figures 3a and 3b). Note that the peak values of southward heat transport are simulated for climates not representing *MEP*: with respect to resolution it is obtained in the T10 simulation (Figure 3a), and for a value of τ_{FRIC} = 1 day with respect to boundary layer friction (Figure 3b). Nevertheless, heat is transported most effectively at *MEP*, leading to warmer polar temperatures. Also, the magnitude and shape of the poleward heat transport variation with latitude at a state of *MEP* concurs well with the observed shape and the annual hemispheric mean magnitude of 3 PW [*Peixoto and Oort*, 1992]. Consequently, the simulated climates at *MEP* have the lowest temperature gradient between the equator and the pole in the lower atmosphere, with poles generally being substantially warmer and tropical areas generally being cooler (Figures 4a and 4b) than the non-*MEP* climates. The smaller differences in the equator-pole temperature gradient shown in Figure 4 are consistent with the more

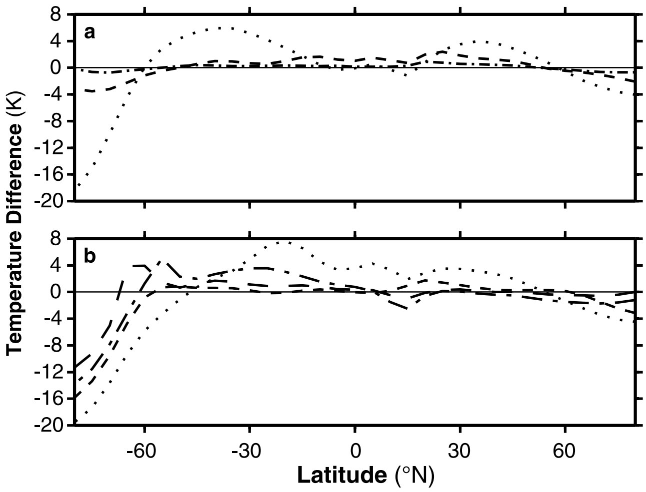

Figure 4. Differences in the latitudinal variation of temperatures for the lowest atmospheric model layer in comparison to the simulated climate of maximum entropy production. (a) effects of different model resolutions between T10 and T42 resolution (dotted), same for T21 (dashed), and T31 (dash-dotted) resolution, each with optimum values of τ_{FRIC}. (b) effects of different intensities of boundary layer turbulence between τ_{FRIC} = 0.1 day and τ_{FRIC} = 3 days (dotted), same for τ_{FRIC} = 1 day (short dashes), same for τ_{FRIC} = 10 days (long dashes), same for τ_{FRIC} = 100 days (dash-dotted), each at T42 resolution.

effective heat transport (Figure 3) and increased rates of entropy production shown in Figure 1.

4. Discussion and Conclusion

[11] We showed that in comparison to the simulated atmospheric circulation at a state of *MEP*, any other model simulation that we conducted led to a weaker effective heat transport to the poles and a greater equator-pole temperature gradient. The results stress the importance of using a sufficiently high model resolution to adequately simulate poleward atmospheric heat transport for the present-day climate. High model resolution allows for sufficient degrees of freedom so that the atmospheric circulation can adjust to such modes that overall entropy production is maximized. For present-day conditions, the minimum resolution is the commonly used T42 resolution. Below this resolution, atmospheric dynamics are too constrained by the reduced spatial resolution, leading to less mid-latitude eddy activity and entropy production, that is, the conversion of APE to KE is less efficient. When the degrees of freedom are not explicitly simulated, as it is the case for boundary layer turbulence, the model parameterization can be tuned to represent a state of *MEP*. We showed that this optimization leads to a reasonable value for the friction time constant and magnitude and pattern of poleward heat transport. However, with the model used it is not possible to show that boundary layer turbulence would actually adjust to the *MEP* state because the simple formulation used in the model does not resolve the detailed dynamics of boundary layer turbulence.

[12] If we assume that the atmospheric circulation is characterized by a state of *MEP* not just for present-day conditions, but also under global change, then these results have far reaching implications for our understanding of global climatic change. For instance during periods with high carbon dioxide concentration during the Cretaceous or the Eocene, the contrast in radiative forcing between the equator and the pole is reduced. It is generally known that GCMs tend to overestimate the equator-pole temperature gradient for those conditions [*Pierrehumbert*, 2002]. This general bias in GCMs may simply be attributed to the fact that the simulated atmospheric circulation does not represent a state of *MEP* under altered climatic forcing, either by not using a sufficiently high model resolution or by not adjusting the parameterization of boundary layer turbulence to represent *MEP*. Consequently, a GCM simulation would underestimate poleward atmospheric heat transport in comparison to the *MEP* state, resulting in an enhanced equator-pole temperature gradient. This suggests that GCMs in general have a fundamental tendency to underestimate poleward atmospheric heat transport. This line of argument also applies to climate model simulations of anthropogenic climatic change, suggesting that changes in atmospheric heat transport, and warming in polar regions, are possibly underestimated.

[13] **Acknowledgments.** AK is grateful for partial support by the general research board of the University of Maryland and the National Science Foundation through grant ATM336555. KF, TK, and FL acknowledge partial support for this study by the Deutsche Forschungsgemeinschaft. We thank Ute Luksch for helpful comments on the manuscript and Garth Paltridge and Ralph Lorenz for their constructive reviews.

References

Dewar, R. C., Information theory explanation of the fluctuation theorem, maximum entropy production, and self-organized criticality in non-equilibrium stationary states, *J. Physics A*, *36*, 631–641, 2003.
Fraedrich, K., C. Ziehmann, and F. Sielmann, Estimates of spatial degrees of freedom, *J. Clim.*, *8*, 361–369, 1995.
Fraedrich, K., E. Kirk, and F. Lunkeit, Portable Univ., Model of the Atmosphere, *DKRZ Report 16*, German Climate Computing Center, Hamburg, Germany, 1998.
Grassl, H., The climate at maximum entropy production by meridional atmospheric and oceanic heat fluxes, *Q. J. R. Meteorol. Soc.*, *107*, 153–166, 1981.
Held, I. M., and M. J. Suarez, A proposal for the intercomparison of the dynamical cores of atmospheric general circulation models, *Bull. Am. Meteorol. Soc.*, *75*, 1825–1830, 1994.
Hoskins, B. J., and A. J. Simmons, A multi-layer spectral model and the semi-implicit method, *Q. J. R. Meteorol. Soc.*, *101*, 637–655, 1975.
James, I. N., and L. J. Gray, Concerning the effect of surface drag on the circulation of a planetary atmosphere, *Q. J. R. Meteorol. Soc.*, *112*, 1231–1250, 1986.
James, I. N., and P. M. James, Ultra-low-frequency variability in a simple atmospheric circulation model, *Nature*, *342*, 53–55, 1989.
Jaynes, E. T., Information theory and statistical mechanics, *Phys. Rev.*, *106*, 620–630, 1957.
Lorenz, E. N., Generation of available potential energy and the intensity of the general circulation, in *Dynamics of Climate*, edited by R. C. Pfeffer, pp. 86–92, Pergamon Press, Oxford, UK, 1960.
Lorenz, R., Full Steam Ahead - Probably, *Science*, *299*, 837–838, 2003.
Lorenz, R. D., J. I. Lunine, P. G. Withers, and C. P. McKay, Titan, Mars and Earth: Entropy production by latitudinal heat transport, *Geophys. Res. Lett.*, *28*, 415–418, 2001.
Ozawa, H., A. Ohmura, R. D. Lorenz, and T. Pujol, The second law of thermodynamics and the global climate system - A review of the maximum entropy production principle, *Rev. Geophys.*, in press, 2003.
Paltridge, G. W., Global dynamics and climate - a system of minimum entropy exchange, *Q. J. R. Meteorol. Soc.*, *101*, 475–484, 1975.
Paltridge, G. W., The steady-state format of global climate, *Q. J. R. Meteorol. Soc.*, *104*, 927–945, 1978.
Peixoto, J. P., and A. H. Oort, *Physics of Climate*, American Physical Society (New York), 1992.
Pierrehumbert, R. T., The hydrologic cycle in deep-time climate problems, *Nature*, *419*, 191–198, 2002.

A. Kleidon, Department of Geography, University of Maryland, 1159 Lefrak Hall, College Park, MD 20742, USA. (akleidon@umd.edu)
K. Fraedrich, T. Kunz, and F. Lunkeit, Meteorologisches Institut, Universität Hamburg, Bundesstraße 55, 20146 Hamburg, Germany. (fraedrich@dkrz.de; kunz@dkrz.de; lunkeit@dkrz.de)