On Single Station Forecasting: Sunshine and Rainfall Markov Chains

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Abstract:

Probabilities of weather states are predicted by first order Markov chains using single station data only. Two examples of weather phenomena are discussed which are particularly suited for short range forecasting: daily sunshine measurements and the rainfall combined with three hourly past weather observations at Berlin-Dahlem. Two probability forecasts are distinguished, both of which start from an initial observation: (i) The chance of a weather state to occur at a prescribed time step; this forecast tends towards the local climate defined by the relative frequency this weather state is occupied with. (ii) The probability of a weather state to occur (at least once) within a prescribed time interval. For increasing interval length this forecast tends towards probability one. Finally, it is shown, how the diurnal cycle can be included in the Markov chain forecasts. Before forecast and forecast skill evaluation the procedure of model building requires a careful calibration of the finally used model version. This implies a thorough statistical analysis of theoretical and empirical probability distributions.

Zusammenfassung: Zur stochastischen Stationsvorhersage (1. Teil): Markov-Ketten von Sonnenschein und Regen

Die Wahrscheinlichkeit von Wetterzuständen wird mit Hilfe der Markov-Ketten erster Ordnung vorhergesagt, wobei nur die Daten einer Station Verwendung finden. Zwei Beispiele von Wetterphänomenen werden ausgewählt, die relevant für die Kurzfrist-Vorhersage sind: tägliche Sonnenschein-Messungen und der Niederschlag im Zusammenhang mit den dreistündigen Wetterbeobachtungen in Berlin-Dahlem. Zwei Arten der Wahrscheinlichkeitsvorhersage werden unterschieden, die beide von Anfangsbeobachtungen ausgehen: (i) Die Wahrscheinlichkeit eines Wetterzustandes zu einem vorgeschriebenen Zeitschritt; diese Vorhersage strebt dem lokalen Klima zu, das durch die relative Häufigkeit des Wetterzustandes bestimmt ist. (ii) Die Wahrscheinlichkeit eines Wetterzustandes (wenigstens einmal) in einem vorgeschriebenen Zeitintervall aufzutreten. Mit zunehmender Intervall-Länge strebt diese Vorhersage zur Wahrscheinlichkeit eins. Schließlich wird gezeigt, wie der Tagesgang in eine Markov-Kettenvorhersage einbezogen werden kann. Bevor Vorhersagen gemacht und deren Güte ausgewertet werden können, erfordert die Konstruktion des Modells eine sorgfältige Eichung der endgültigen Modellversion. Das schließt eine statistische Analyse theoretischer und empirischer Wahrscheinlichkeitsverteilungen ein.

Résumé: Sur la prévision stochastique à une seule station (1e partie): Chaînes de Markov de l'insolation et précipitation

On prédit la probabilité d'états métérologiques à l'aide de chaînes de Markov du premier ordre, en utilisant les données d'une seule station. On envisage deux exemples de phénomènes, qui sont particulièrement importants pour la prévision à brève échéance: les mesures journalières de l'insolation et les précipitations, en liaison avec les observations trihoraires à Berlin-Dahlem. On distingue deux sortes de prévision de probabilité, qui partent toutes deux d'observations initiales: (I) la probabilité d'un état météorologique à un pas de temps prescrit; cette prévision tend vers le climat local, défini par la fréquence relative de cet état; (II) la probabilité pour un état de se réaliser (au moins une fois) au cours d'un intervalle de temps préscrit; pour un intervalle croissant, la probabilité tend vers un. Finalement, on montre comment la cycle diurne peut être inclus dans les prévisions par une chaîne de Markov. Avant qu'on puisse faire une prévision et estimer sa valeur, la construction d'un modèle requiert un étalonnage soigneux de la version finalement adoptée. Ceci implique une analyse statistique complète des distributions de probabilité théoriques et empiriques.

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1 Introduction

Weather fluctuations are an essential part of stochastic climate models, in which they are parameterized by simple additive stochastic processes of very short time scale. Magnitude and structure of the resulting climatic response on these weather fluctuations show reasonably good agreement with observations; i.e. the incorporation of simply parameterized stochastic weather fluctuations helps to explain climate variations. The question remains whether these stochastically parameterized weather disturbances can describe real weather data and, if so, whether (and how far) these parameterizations are applicable to short range weather prediction. In two papers we shall be concerned with an answer of this question and, thereby, confine ourselves to single station weather observations. Generally, such weather variables are measured in discrete time steps; the observed variables, however, may be discrete (e.g. classes or states of rainfall or cloudiness) or continuous (e.g. pressure). Accordingly, two different classes of stochastic point processes in discrete time must be selected which depend on weather variables being continuous or discrete.

Three basic weather variables are selected for our study of stochastic single station prediction. They are related with the two classes of stochastic point processes and will be discussed in two separate parts:

This paper (part one) describes sunshine and rainfall. These weather variables quantify mesoscale phenomena. Realizing their randomness and discrete nature, not the exact state variable but the probability of a discrete interval is the appropriate predictand. Markov chains are chosen as prediction models to provide probability but not state variable forecasts. Such models have been successfully applied to rainfall simulation since the 60's (e.g. GABRIEL and NEUMANN, 1962). The following section reviews the theoretical background; Markov chains, some related distributions and probability forecasts are briefly discussed, to derive a linear, stochastic weather prediction model for the single station Berlin-Dahlem. The appropriate sunshine and rainfall prediction schemes are deduced in Sections 3 and 4, respectively. These forecast models are calibrated by the time history of the local weather data and evaluated to study the feasibility of probability forecasts based on single station observations.

In another paper (FRAEDRICH and DÜMMEL, 1983) the geopotential is described as an essentially synoptic scale variable. The geopotential trajectory in time defines troughs and ridges passing over a station. It is not so much the exact local geopotential amplitude (and level), which is of forecast value, but the qualitative behaviour predicting the lead time of highs and lows. Gaussian processes in terms of autoregressive or moving average models will be applied; they have been successfully used to predict economic time series (BOX and JENKINS, 1976) and to simulate some aspects of long range weather variations (KATZ and SKAGGS, 1981).

2 Basic concepts

2.1 Markov chains repeated

A discrete point process $X_t(i)$ or $X_t = i$ can be defined for discrete states i (or j) = 1,..., s occurring at discrete time steps t = 1,...,n (or m). In the theory of Markov chains the observed event $X_t(i)$, which can be regarded as the outcome of a trial at time t, depends on the outcome of the directly preceeding trial (t − 1). Thus, the outcome is not associated with a fixed probability, prob $\{X_t = i\}$, but with a pair corresponding to a conditional probability p_{ij}:

$$p_{ij} = \text{prob}\,\{X_t = j \mid X_{t-1} = i\}$$

In the following section some basic information on first order Markov chains, which are ergodic and finite, will be briefly summarized (FERSCHL, 1970; KEMENY and SNELL, 1976).

A Markov chain consists of a transition probability matrix (p_{ij})

$$(p_{ij}) = \begin{pmatrix} p_{11} & p_{12} & \cdots \\ p_{21} & p_{22} & \cdots \\ \vdots & \vdots & \end{pmatrix} \tag{2.1}$$

mapping an initial state probability vector (a_i) linearly into a predicted state probability vector (a_j):

$$(a_j) = (a_i) \cdot (p_{ij}) \tag{2.2}$$

A probability vector (a_i) defines a probability distribution. The vector components correspond to discrete classes or states $i = 1,\ldots,s$ of the probability distribution; they are non-negative and sum up to unity:

$$\sum_{i=1}^{s} a_i = 1 .$$

In applications it often happens that all components of an initial probability vector, e.g. $(a_i) = (0,0,1,\ldots)$, vanish except the initially observed state.

The transition probabilities p_{ij} are conditional probabilities for the state j following the state i after a one step transition; all possible states j combined follow a state i with probability $\Sigma_j \, p_{ij} = 1$; i.e. (p_{ij}) is a stochastic matrix; a double stochastic matrix has unit sums for both row and column vectors.

If the one step ahead probability forecast is successively repeated, one obtains the n-step or lead time n forecast for a probability vector $(a_j)^{(n)}$ or its element $a_i^{(n)}$, respectively:

$$(a_j)^{(n)} = (a_i)\,(p_{ij})^{(n)} \quad \text{or} \quad a_j^{(n)} = \Sigma\, a_i \, p_{ij}^{(n)} \tag{2.3}$$

Powers of a transition matrix $(p_{ij})^{(n)}$ remain stochastic matrices; their algebraic treatment depends on powers of eigenvalues λ:

$$(p_{ij})^{(n)} = \lambda_0^n \, C_0 + \lambda_1^n C_1 + \ldots$$

which yield $|\lambda| \leqslant 1$ for stochastic matrices and can be ordered according to their magnitude, $1 = \lambda_0 > |\lambda_1| > \ldots$; the matrices, $C_0, C_1 \ldots,$ are defined by the left and right eigenvectors. The matrix C_0 associated with $\lambda_0 = 1$, and the second largest eigenvalue, $|\lambda_1| < 1$, are of further interest.

The ergodic Markov chain settles down to a condition of statistical equilibrium after a sufficiently long lead time $(n \to \infty)$. The equilibrium state probability vector or equilibrium state occupation probability, $(\pi_i) = (\pi_1, \pi_2, \ldots, \pi_s)$, which an ergodic chain approaches, cannot be left again, $(\pi_i) \cdot (p_{ij}) = (\pi_j)$, and is independent of initial conditions a_i:

$$\lim_{n \to \infty} (a_j)^{(n)} = (\pi_j) \tag{2.4}$$

The related equilibrium transition matrix, C_0, is composed of the equilibrium state vector (π_i):

$$\lim_{n \to \infty} (p_{ij})^{(n)} = C_0 = \begin{pmatrix} \pi_1, \pi_2, \ldots \\ \pi_1, \pi_2, \ldots \\ \cdot \quad \cdot \\ \cdot \quad \cdot \\ \cdot \quad \cdot \end{pmatrix} \tag{2.5}$$

This guarantees independence of initial conditions, $(\pi_j) = (a_i) \cdot C_0$. The equilibrium is approached as the powers of the eigenvalues tend to zero. A time scale, τ, can be defined by the largest eigenvalue, $|\lambda_1| < 1$, in terms of an e-folding equivalent to the geometrically decreasing powers, $\lambda_1^{\tau} = e^{-1}$ or

$$\tau = 1/\ln \lambda_1^{-1} \tag{2.6}$$

2.2 Some probability distributions

Probability distributions, P, related to discrete point processes $X_t(i)$ can be evaluated empirically from the observed time series $P(X_t) = \text{prob}\{X_t\}$, and derived for Markov chains from the estimated transition probabilities.

Period lengths, first passage and recurrence times, and occupation times are defined in the following:

Period length: The probability distribution $P(L_i = n)$ of n successive time steps, $L_i = n$, during which the observed process, $X_t = i$, remains in one and the same state i before leaving to another is formally defined by

$$P(L_i = n) = \text{prob}\{X_{m+t} = i \text{ for } 0 < t < n-1,\ X_{m+n} \neq i \mid X_{m-1} \neq i\} \qquad (2.7a)$$

The expectation or mean values, $E_i(L_i)$, are defined conditional on starting from state i:

$$E_i(L_i) = \sum_{n=1}^{\infty} n\, P(L_i = n)$$

For first order Markov chains one obtains a geometric distribution for the period length of state i:

$$P(L_i = n) = p_{ii}^{n-1}(1 - p_{ii}) \qquad (2.7b)$$

with mean $E_i(L_i) = (1 - p_{ii})^{-1}$ and variance $\text{Var}(L_i) = p_{ii}(1 - p_{ii})^{-2}$. Period lengths of zero order Markov chains can be defined by the equilibrium state probability ($p_{ii} = \pi_i$). In general, they deviate strongly from the observed (2.7a) and first order Markov chain periods (2.7b).

First passage time: Conditional on state i being occupied initially, the probability, $P(T_j = n)$, that the state j occurs at time $T_j = n$ and is avoided at times $1 \leqslant t \leqslant n-1$, is called first passage probability from state i to state j:

$$P(T_j = n) = \text{prob}\{X_{m+t} \neq j \text{ for } 0 \leqslant t \leqslant n-1;\ X_{m+n} = j \mid X_m = i\} \qquad (2.8a)$$

The first passage probability $P(T_j = n) = f_{ij}$ for Markov chains can be evaluated successively from the transition probabilities p_{ij}

$$p_{ij}^{(n)} = \sum_{r=1}^{n} f_{ij}^{(r)}\, p_{jj} \qquad (2.8b)$$

starting at the first step, n = 1, with $p_{ij} = f_{ij}$. The mean first passage time $E_i(T_j) = m_{ij}$ and its variance can be deduced in matrix form (KEMENY and SNELL, 1976):

$$(m_{ij}) = (I - Z + E\, Z_{dg})\, D$$

with the fundamental matrix $Z = (I - P + C_0)^{-1}$, the diagonal matrix Z_{dg} of Z, the matrix D with only diagonal elements $d_{jj} = \pi_j^{-1}$, the unit matrix I, the matrix E with all entries 1.

A special case is defined by the return probability $P(T_j = n) = f_{jj}^{(n)}$; the related mean recurrence time from state i to state j gives

$$E_j(T_j) = m_{jj} = \pi_j^{-1}$$

which is identical to the inverse of the equilibrium state occupation probability.

Occupation time: The total number N of time steps a stochastic process $X_t(j)$ of $1 \leqslant t \leqslant m$ steps spends in state i defines the total occupation time

$$N_m(j) = \sum_{t=1}^{m} \zeta_t(j) \qquad (2.9a)$$

where the counting variable $\zeta_t(j) = 1$ or 0 for $X_t = j$ or $= i$. A mean occupation time, $E_i(N_m(j)) = 0_{ij}$, of state j conditional on starting at state i can be defined for Markov chains:

$$0_{ij} = \sum_{t=1}^{m} E_i(\zeta_t(j)) = \sum_{t=1}^{m} p_{ij}^{(t)} \tag{2.9b}$$

If related to first passage and recurrence times, additional information can be obtained. While the stochastic process X_t passes from state i to, say, state k, the occupation times of state j between the states i and k can be evaluated applying the results of absorbing Markov chains (a state k which once entered cannot be left, is classified absorbing with transition probability $p_{kk} = 1$). While the Markov chain passes from the initial state i to an absorbing state k, the average number (and variance) of time steps occupied by state j yields in matrix form (KEMENY and SNELL, 1976):

$$(0_{ij}) = (I - Q)^{-1}$$

where Q is a submatrix concerning all states i and j, but the absorbing state k and I is the identity matrix.

A special case evolves in connexion with the average recurrence time $m_{ii} = \pi_i^{-1}$; during the time interval n a return to state i should occur n/m_{ii}-times. Thus, the average occupation time of the state j between the states i (i.e. during passage or return from state i to state i) simply yields $\pi_j/\pi_i = (n/m_{jj})/(n/m_{ii})$.

2.3 Probability forecasting and its verification

In this paper Markov chain models are applied to yield probability forecasts of weather elements. At least two forecast objectives may be distinguished: The probability of an event to occur at a certain future time (point forecast) or within a future time interval (integral forecast):

Point or time step forecasts: Given the initial probability distribution vector (a_i) at time step $n = 0$; the probability $F(j)$ for a state j to occur at the n-th step ahead is given by the j-th element of the probability vector $(a_j)^{(n)}$:

$$F(j) = a_j^{(n)} = \sum_i a_i p_{ij}^{(n)} \tag{2.10}$$

For increasing time steps ($n \to \infty$) the point forecasts of a state, $a_j^{(n)}$ approach the equilibrium or climate probability π_j.

Integral or time interval forecasts: Given the initial distribution vector (a_i) at time step $n = 0$; the probability, $F(j)$, for a first passage to the state j occurring within the next n steps following the initial state is given by j-th element the probability vector $(A_j)^{(n)}$:

$$F(j) = A_j^{(n)} = \sum_{t=1}^{n} a_i f_{ij}^{(t)} \tag{2.11}$$

Thus, the probability of a first passage from state i to state j within the time interval n approaches the value one after $n \to \infty$ time steps.

If diurnal cycles are important, they can be incorporated by transition matrices fixed at the daily observation hours; the related conditional probabilities are determined by pairs of consecutive events at these fixed hours. Point and integral forecasts are performed accordingly.

The forecasts have to be verified by observations. A common verification score of probability forecasts is the so-called half Brier score, B, of $1 \leqslant t \leqslant n$ probability forecasts (BRIER, 1950):

$$B = n^{-1} \sum_{t=1}^{n} (F_t(j) - \delta_t(j))^2 \tag{2.12}$$

where $F_t(j)$ is the forecast probability of one individual state j and $\delta_t(j)$ is either one or zero depending on whether the individual state is or is not observed. As probability forecasts of the remaining (i.e. complementary) states $(1 - F_t(j))$ are excluded, the half Brier score, B, ranges from zero to one with zero indicating perfect forecasting. It should be noted that, besides quadratic scoring rules, logarithmic or spherical scoring rules can be applied.

The Brier score can be extended to several or all components of the predicted state probability vector. If the state vector as a whole is considered, the forecast probability $F_t(j)$ is compared with the observed probability $\delta_t(j) = 1$. Thus, the resulting Brier score yields a forecast quality measure of the complete model, which is not confined to selected states.

Forecast probabilities F of prediction models generally depend on the forecast lead time n. For Markov chains, which are considered here, the forecast probability $F_M(j)$ $(= F_t(j))$ is defined for point or integral forecasts, respectively:

$$F_M(j) = a_j^{(n)} \text{ or } A_j^{(n)} \tag{2.13}$$

Generally point forecasts are verified. Verification scores of lead time dependent probability forecasts are often compared with reference models which provide lead time independent predictions. Three reference schemes are summarized:

(i) Climate or equilibrium predictions are defined by the transition matrix C_0:

$$F_c(j) = \pi_j \tag{2.14}$$

which is composed of the observed climate or state occupation probabilities π_j.

(ii) Persistence forecasts are made by the identity matrix, where both the initial and predicted state concur with probability one:

$$F_p(i) = F_p(j) = 1 \tag{2.15}$$

(iii) Random forecasts yield equal probability for all states to be predicted:

$$F_r(j) = (\text{number of states predicted})^{-1} \tag{2.16}$$

i.e. the transition probability matrix for random forecasts is composed of the equilibrium state occupation probabilities of a double stochastic transition matrix.

Markov probability forecasts tend to the observed equilibrium (or climate) state occupation probabilities (π_i) with increasing lead time. Therefore, it is reasonable to define a skill score, S_c, which relates both Markov and equilibrium or climate forecasts:

$$S_c = \frac{B_c - B_M}{B_c} \tag{2.17}$$

B_M is the Brier score for Markov prediction and B_c is the Brier score using the equilibrium or climate state probabilities as forecasts. A perfect skill score is 1, zero indicates no skill, negative skills are possible. Skill scores, S_p, and S_r, related to persistence and random forecasts, can be defined analoguously.

2.4 Model building

The procedure to fit a stochastic model to a data set consists of three steps (BOX and JENKINS, 1976; KASHYAP and RAO, 1976), which follow the theoretical outline given above (Section 2.1 to 2.3). A priori, the state variables have to be defined, i.e. the type, i, and number, s, of states to be predicted. The model building scheme, described in this section, is prosecuted afterwards:

(i) Identification and estimation of the model: The identification technique determines whether the model type (e.g. the Markov chain) fitting the data, is an appropriate choice. Therefore, model parameters have to be estimated in advance to compare a variety of models, which are distinguished by their order.

The question whether or not the observed data sequence can be represented by a r-th order Markov chain of an s-state process is answered by several tests, which have the following procedure in common. The order r should be chosen sufficiently large to include any reasonable model. Then it is possible to test successfully lower orders until an order is reached below which it is unreasonable to go.

One test is based on Akaike's information criterion (AIC) by minimizing the AIC estimate (see GATES and TONG, 1976; EIDSVIK, 1980); it is not the goal of this test to select the model which produces minimum prediction error, but to penalize the error by order (i.e. by twice the number of parameters) to obtain a parsimonious model; i.e. the test procedure attempts to balance overfitting, which requires more parameters, and underfitting, which leads to an increased residual variance.

Another method is based on an application of the chi-squared test (HOEL, 1954; ANDERSON and GOODMAN, 1957; LOWRY and GUTHRIE, 1968). The test statistic Z compares the transition probabilities $p_{i,j,\dots,k,l}$, of higher (i,j, ..., k,l) and lower (j, ..., k, l) order models of s states:

$$Z = \sum_{i,j,\dots,k,l=1}^{s} n_{i,j,\dots,k,l}\,(p_{i,j,\dots,k,l})^2/p_{j,\dots,k,l} \qquad (2.18)$$

with the absolute frequency $n_{i,j,\dots,k,l}$. As Z is asymptomatically chi-squared distributed, the order of the process can be estimated as follows:

(a) A chain of zero order ($(p_{ij}) = C_0$; i.e. the climate or equilibrium transition matrix) describes the data sufficiently when it produces a non-significant test statistic compared with higher order models. Then, predictable Markov properties cannot be extracted from the data.

(b) A chain of high order ($p_{i,j,\dots,k,l}$) describes the data sufficiently, when it produces a significant improvement compared with the lower and a non-significant test statistic compared with the higher Markov chain. Thus, the testing procedure estimates whether the data series includes Markov properties at all (a) and the goodness of fit.

In this paper the latter more traditional method is chosen for testing the order and the goodness of fit of Markov chains. Although test statistics of both methods seem to be similar (instead of the quadratic a logarithmic measure is used), the penalty function proportional to the number of parameters appears somewhat arbitrary. Therefore, the automatic order determination by the AIC procedure is replaced by judging the significance level. Furthermore, the prediction error is preferred to be considered independently of the order test by a problem orientated forecast verification scheme. Finally, confidence intervals are placed on each estimated parameter (transition probability) applying standard methods.

(ii) Model analysis or diagnostic checking ascertain that important model related distributions fit the observations. The chi-squared or Kolmogoroff-Smirnov test can be applied.

(iii) Decision making is the final step to ensure that the model has sufficient quality hindcasting the observations being used to fit the model and, particularly, forecasting independent data. The quality is evaluated by the quadratic Brier scoring rule.

3 The Berlin Sunshine Markov Chain

3.1 The observations: states of sunshine (θ_i)

25 years (1953–1977) of daily sunshine hours, θ_{abs}, provide the basic data set, which is obtained from sunshine recorders (Campbell-Stokes, Stade) at the station Berlin-Dahlem. The day to day θ_{abs}-records are normalized by the astonomically (i.e. maximum) possible sunshine duration D = 2 arccos (− tan ϕ tan Δ) with the latitude ϕ, the declination of the sun Δ and an estimated correction for the horizontal extinction of 1.5 degrees for both sunrise and sunset. This leads to a new time series of the

relative sunshine duration per day, $\theta = \theta_{abs}/D$ (percentage of possible sunshine, or relative sunshine); its complement C_a

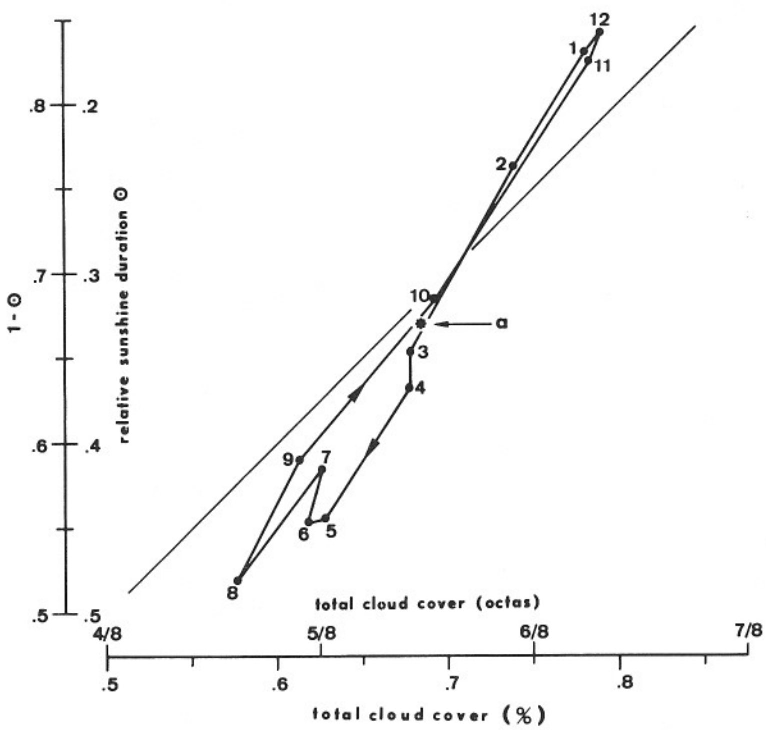
$$C_a = 1 - \theta, \quad 0 \leqslant \theta \leqslant 1$$

is an objectively observed approximation to the daytime average of the areal cloud cover, which is defined by the downward projection of clouds on parallel lines. Synoptic surface observations of the total or point cloud cover, C_c, however, provide a subjective measure ot the converging projection of all clouds in the celestial sphere onto a point, i.e. the surface observer (HOYT, 1977; COURT, 1978; VACOWAR et. al., 1976).

Before building a sunshine Markov chain forecast model, the states to be predicted must be defined. A descriptive statistic, which is discussed in the following, provides the climatologically relevant background information:

(i) Annual cycle (Figure 1): A climograph of monthly means combines the relative sunshine θ (or its complement C_a) and the independently observed total or point cloud cover C_c which is averaged over the synoptic surface observations during daylight hours. In winter months the daily and monthly averaged point cloud cover C_c underestimates the areal cloud cover $C_a = 1 - \theta$, whereas the opposite holds for the summer, so that the annual means almost coincide. This should be realized in the following, when interpreting relative sunshine in terms of cloudines (FOX, 1961).

(ii) Frequency distribution (Figure 2): A seasonal frequency distribution (and its accumulation) of daily records of relative sunshine θ is based on 25 seasons and evaluated for ten θ-classes ($\theta > 0$) of $0.1 \cdot \theta$ interval length, plus a separate (eleventh) class of no sunshine ($\theta = 0$) per day. Additional information is gained from the seasonal sunshine totals and how much each of the ten θ-classes contribute (individually or accumulated) to it:

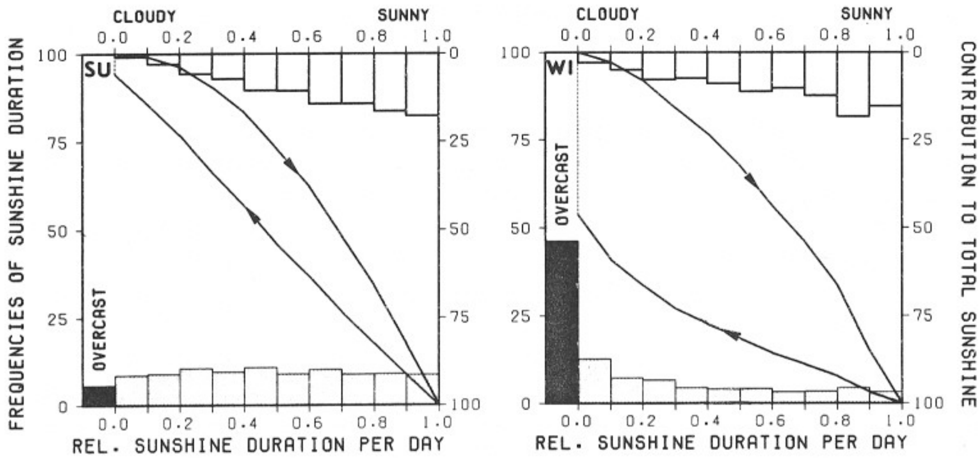


● **Figure 1**

Climograph of relative sunshine duration, θ, (ordinate) and total cloud cover (abscissa) from surface observations averaged over daytime.

● **Bild 1**

Klimograph der relativen Sonnenscheindauer, θ, (Ordinate) und der Gesamtbedeckung (Abszisse), gemittelt über die Tageszeit.



● **Figure 2** Relative and accumulated distributions in relative sunshine classes for summer (left) and winter (right); bottom: relative frequencies; top: contributions to total sunshine hours.

● **Bild 2** Relative und akkumulierte Verteilungen in Klassen der relativen Sonnenscheindauer für Sommer (links) und Winter (rechts); unten: Häufigkeiten; oben: Beiträge zur Sonnenscheindauer.

The relative daily frequencies in summer are almost uniformely distributed over all classes of relative sunshine duration per day, θ, but they exhibit a tendency towards the normal distribution: the class including the average $0.4 < \theta \leqslant 0.5$ occurs most frequently. This is in contrast to the winter season, for which the daily percentage sunshine, θ, is almost exponentially geometrically distributed with about half of all days being completely overcast ($\theta = 0$).

The relative contributions to a season's sunshine total by each θ-class show similar distributions for summer and winter. Not unexpectedly, the few days with much relative sunshine, θ, contribute most of the total. If accumulated, all classes above or below $\theta \sim 0.7$ account for about half of the sunshine hours of a season. Days without sunshine ($\theta = 0$) are, of course, excluded from this statistics.

(iii) Definition of θ_i-states of relative sunshine duration (Table 1): The number of eleven θ-classes of relative sunshine are reduced to four climatologically and statistically meaningful θ (or $C_a = 1 - \theta$) intervals to be predicted. These four states are defined in analogy to the cloud amount categories of MOS forecasts (see e.g. KLEIN, 1982):

$$(\theta_i) = (\theta_1, \theta_2, \theta_3, \theta_4) = (\text{OVC, BKN, SCT, CLR})$$

with overcast (OVC), broken (BKN), scattered (SCT), and clear (CLR) sky conditions.

In Table 1 they are related to the climatologically relevant statistics of relative sunshine duration. It should be noted that the relative and absolute frequencies of the four classes change from summer to winter, shifting the modal (most frequent) values from broken and scattered to overcast. But, contributions of the four states to the total sunshine hours hardly vary from summer to winter (the same holds for spring and fall, which are not considered here): About half of a season's sunshine (52–54%) is observed during scattered skies (θ_3) about half of which (27–29%) occurs during the broken sky states (θ_2), another half (16–18%) comes from the clear sky category (θ_4), and overcast (θ_1) contributes only a negligibly small amount (1–3%) to the total sunshine. Thus, these states, although referring to cloudiness categories and varying in frequency from summer to winter, divide the sunshine totals into four uniquely defined and

■ **Table 1** States of relative sunshine duration, θ, their observed frequencies and contributions to total sunshine hours of mean seasons.

■ **Tabelle 1** Zustände der relativen Sonnenscheindauer, θ, beobachtete Häufigkeiten und Anteil am Gesamtsonnenschein der mittleren Jahreszeiten.

				Summer				Winter			
				frequency		total sunshine hours		frequency		total sunshine hours	
		state	relative sunshine	abs.	rel.	abs.	rel.	abs.	rel.	abs.	rel.
θ_1	OVC	overcast bedeckt	$0 \leqslant \theta < 0.1$	324 d	14%	7 h	1%	1306 d	59%	5 h	3%
θ_2	BKN	broken wolkig	$0.1 \leqslant \theta < 0.5$	910 d	40%	178 h	27%	501 d	22%	43 h	29%
θ_3	SCT	scattered heiter	$0.5 \leqslant \theta < 0.9$	841 d	37%	356 h	54%	344 d	16%	78 h	52%
θ_4	CLR	clear sonnig	$0.9 \leqslant \theta \leqslant 1.0$	200 d	9%	119 h	18%	74 d	3%	24 h	16%
			Σ	2275 d	100%	660 h	100%	2225 d	100%	150 h	100%

seasonally stable classes. In this sense the four states of relative sunshine duration (conveniently attached with cloud cover names) provide a climatologically meaningful measure of the total sunshine! The probabilities of these four sunshine states define a vector of four components, which can be predicted by a Markov chain model. The related transition matrix is estimated by the Berlin sunshine observations of 25 summer (June to August) and winter (December to February) seasons. This leads to a single station probability forecast scheme of relative sunshine for one day time steps (next Section).

3.2 The model; hindcast and forecast

Maximum likelihood estimates of transition probabilities, p_{ij}, are obtained from normalized joint frequency distributions of present (t) and preceeding (t − 1) sunshine states (Table 2). The seasons are defined to coincide with the beginning and the end of a sunshine state period, which must not be interrupted by calendar date. This leads to the small differences in the equilibrium or climate state occupation frequencies depending on whether the summation is taken over past (t − 1) or present (t) days.

(i) Model identification and estimation (Table 2): The model identification and estimation procedure (Section 2.4) leads to a first order Markov chain (Section 2.1) for both summer and winter seasons. The chi-squared and AIC tests lead to the same conclusion; furthermore, forecasts with higher order chains did not improve the skill considerably. The estimated transition probability matrices are discussed in the following. They include 95% confidence limits for the parameters and exhibit great differences between summer and winter seasons:

In summer, cloudy weather has the tendency to improve; with 50% probability an overcast day (t − 1: θ_1 = OVC) will be followed by a broken sky day (t: θ_2 = BKN). The states of broken and scattered cloudiness tend to persist with high probabilities of about 50%, whereas a clear sky day (t − 1: θ_4 = CLR) is more frequently followed by a scattered (t: θ_3 = SCT) than by another clear sky day. This reflects the enhanced chances of shower activity (or their nocturnal left over) on days following a clear or sunny day.

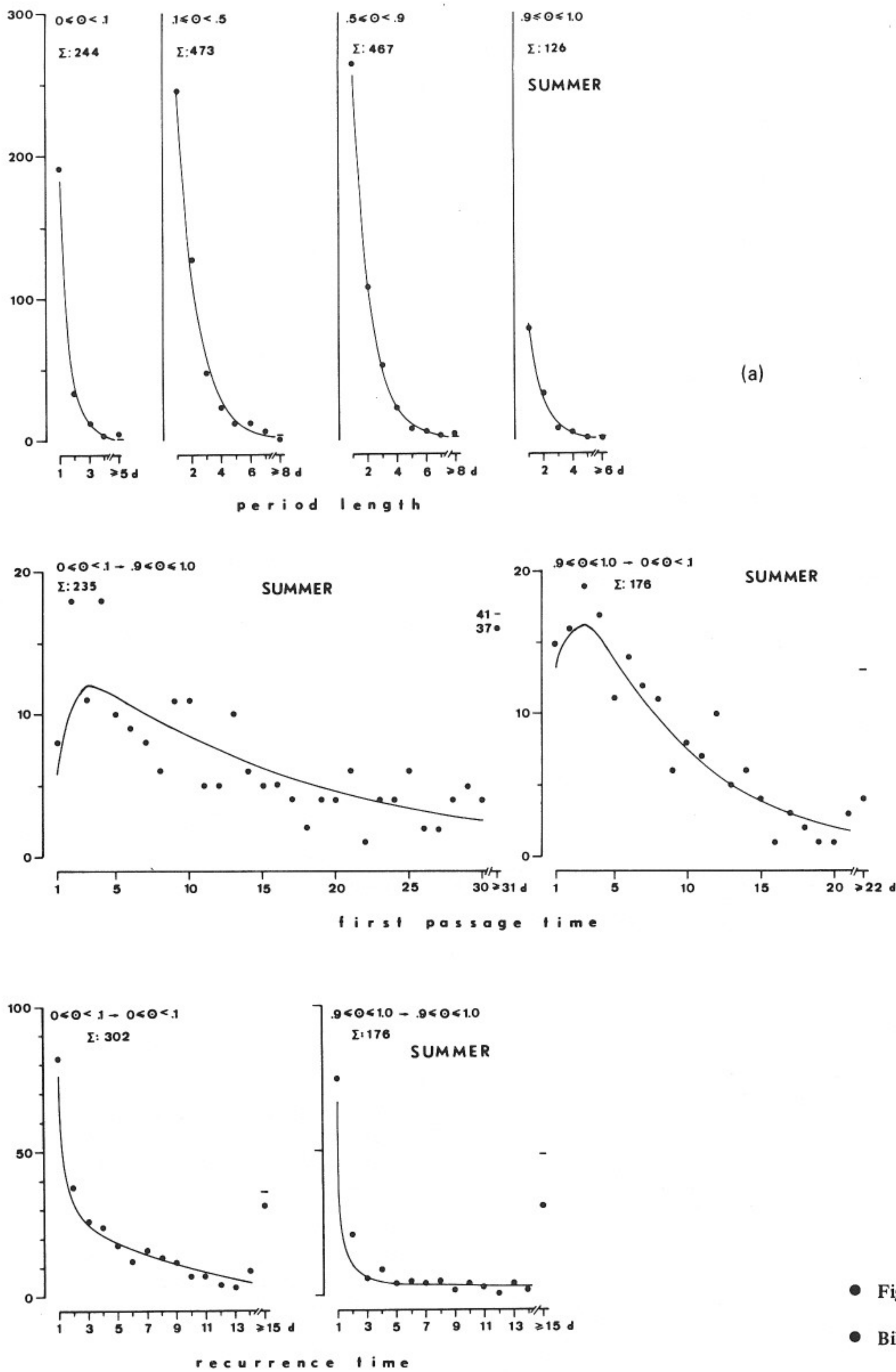
■ **Table 2** Estimated joint frequency distributions and transition probabilities for present (t) and preceeding (t−1) sunshine states (θ_i). Bottom: equilibrium (climate) state occupation probabilities.

■ **Tabelle 2** Geschätzte Häufigkeitsverteilungen und Übergangswahrscheinlichkeiten zwischen gegenwärtigen (t) und vorangegangenen (t−1) Zuständen der relativen Sonnenscheindauer (θ_i). Untere Reihe: Gleichgewichts-(Klima-) Wahrscheinlichkeiten der Besetzung von Zuständen θ_i.

relative Sonnenscheindauer – relative sunshine duration												
Sommer Summer		Häufigkeiten – frequencies					P_{ij}:	Übergangswahrscheinlichkeiten ± 95% Konfidenz transition probabilities ± 95% confidence limits				
t−1 \ t		Zustand – state						Zustand – state				
	i:	j: 1	2	3	4	Σ	j:	1	2	3	4	Σ
OVC	1	82	161	74	8	325		.252 ± .047	.495 ± .054	.228 ± .045	.025 ± 0.16	1.
BKN	2	142	437	298	26	903		.157 ± .023	.484 ± .032	.330 ± .030	.029 ± .010	1.
SCT	3	85	291	381	91	848		.100 ± .020	.343 ± .031	.450 ± .033	.107 ± .020	1.
CLR	4	15	21	88	75	199		.075 ± .036	.106 ± .042	.442 ± .069	.377 ± .067	1.
Klima climate		324	910	841	200	2275	π_j:	.142 ± .014	.400 ± .020	.370 ± .019	.088 ± .011	1.

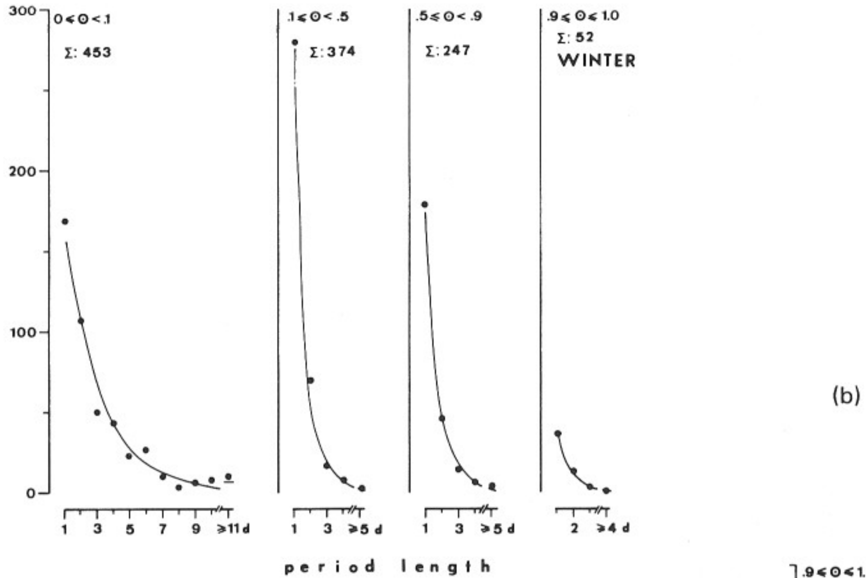
Winter		Häufigkeiten – frequencies					P_{ij}:	Übergangswahrscheinlichkeiten ± 95% Konfidenz transition probabilities ± 95% confidence limits				
t−1 \ t		Zustand – state						Zustand – state				
	i:	j: 1	2	3	4	Σ	j:	1	2	3	4	Σ
OVC	1	859	285	148	15	1307		.657 ± .025	.218 ± .022	.113 ± .017	.012 ± .005	1.
BKN	2	278	131	78	12	499		.557 ± .043	.262 ± .038	.156 ± .031	.024 ± .013	1.
SCT	3	149	70	101	25	345		.431 ± .052	.203 ± .042	.293 ± .048	.073 ± .027	1.
CLR	4	20	15	17	22	74		.270 ± .101	.203 ± .091	.230 ± .095	.297 ± .104	1.
Klima climate		1306	501	344	74	2225	π_j:	.587 ± .020	.225 ± .017	.155 ± 0.15	.033 ± .007	1.

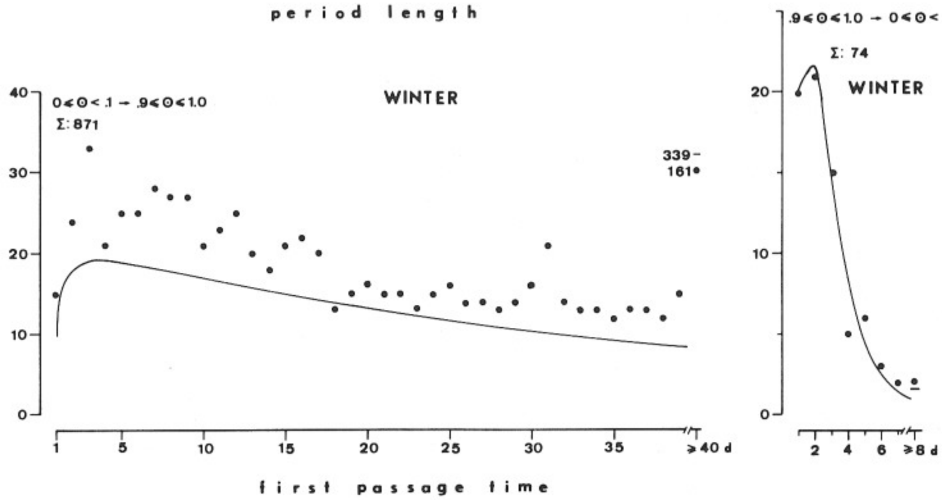
In winter, as expected, the most frequently occupied overcast state (OVC) is the main attractor for all predictions; e.g. an overcast day ($t-1: \theta_1$ = OVC) is followed by another overcast day ($t: \theta_1$ = OVC) with 65% probability. Additionally, all states inhibit the tendency of persistence as a secondary attractor, which is indicated by the magnitude of the diagonal elements of the transition probability matrix. The tendency towards persistence grows when states of enhanced relative sunshine duration occur: A clear day ($t-1: \theta_4$ = CLR) is followed by another clear day ($t: \theta_4$ = CLR) with 30%, by an overcast day ($t: \theta_1$ = OVC) with 27% probability; the other two states share the remaining probabilities. As these transition probabilities are almost equally distributed between 20% and 30%, probability forecasts from a clear winter day to the next day are nearly random; i.e. for this initial condition both the persistence and the overcast attractors have strong influence on the prediction. The Markov chain forecasts approach the climate states (π_i) after a few time steps. The related e-folding time scale τ (2.6) yields $\tau \sim 1.1$ d for summer and 0.9 d for winter seasons.

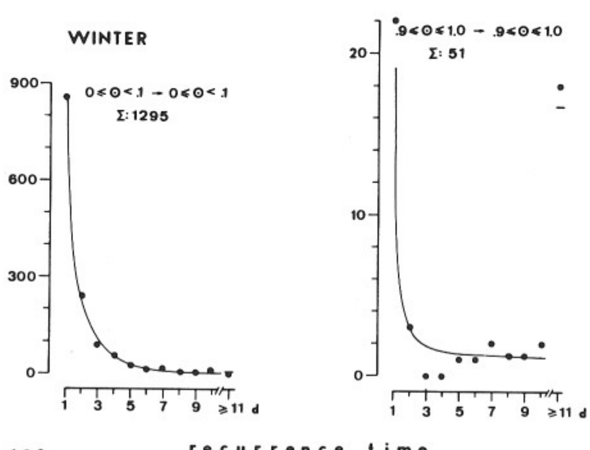


● Figure 3a

● Bild 3a







- **Figure 3a and b**

  Empirical (dotted) and Markov chain (full line) distributions of period length (top), first passage time (middle), and recurrence time (bottom) for selected sunshine states θ_i.

- **Bild 3a und b**

  Beobachtete (Punkte) und Markov-Ketten (durchzogene Linie) Verteilungen der Periodenlänge (oben), der Durchgangszeit (Mitte) und der Rückkehrzeit (unten) für ausgewählte Zustände, θ_i, der relativen Sonnenscheindauer.

■ **Table 3** Average first passage times and period lengths of the sunshine Markov chain and of the observations (in brackets). The diagonal values are mean recurrence times π_j^{-1}.

■ **Tabelle 3** Mittlere Durchgangszeiten und Periodenlängen der Sonnenschein-Markov-Kette und der Beobachtungen (in Klammern). Die Diagonalelemente sind mittlere Rückkehrzeiten π_j^{-1}.

Summer		Mittlere Durchgangszeit in Tagen mean first passage time in days				mittlere Perioden mean periods
nach to von from		Zustand – state				
	j:	1	2	3	4	L
i:						
OVC	1	7.0(6.0)	2.4(2.6)	3.5(3.7)	18.0(16.7)	1.3(1.3)
BKN	2	7.8(6.4)	2.5(2.5)	3.1(3.1)	17.7(16.4)	1.9(1.9)
SCT	3	8.4(7.3)	3.0(3.1)	2.7(2.6)	16.2(14.1)	1.8(1.8)
CLR	4	8.9(7.5)	4.0(4.3)	2.6(2.4)	11.4(7.8)	1.6(1.6)
					π_j^{-1}	

Winter		Mittlere Durchgangszeit in Tagen mean first passage time in days				mittlere Perioden mean periods
nach to von from		Zustand – state				
	j:	1	2	3	4	L
OVC	1	1.8(1.7)	4.6(5.2)	7.9(8.0)	42.3(24.0)	2.9(2.9)
BKN	2	1.9(2.0)	4.5(4.2)	7.6(7.3)	41.6(21.8)	1.4(1.4)
SCT	3	2.3(2.4)	4.8(5.3)	6.4(6.1)	39.1(20.2)	1.4(1.4)
CLR	4	2.7(2.8)	4.8(5.5)	6.6(6.1)	30.0(12.5)	1.4(1.4)
					π_j^{-1}	

(ii) Model analysis (Figure 3, Tables 3 and 4): Distributions, means and variances of meteorologically relevant time or period scales (Section 2.2) are compared between model and observations. This includes the period length of states, and the first passage and recurrence times for the extreme states overcast, OVC, and clear, CLR (Figure 3, Table 3). The hypothesized similarity between model and observations can be accepted for almost all distributions by visual inspection and, with 95% significance, by a chi-squared test. The only exceptions are the winter first passage times to the clear state $(0.9 < \theta \leqslant 1.0)$; the Markov chain tends to underestimate the frequency of short first passage times, and vice versa.

A one day period length (2.7) occurs most frequently for all states in both Markov and observed distributions; the observed frequencies reveal a rapid geometric decrease with increasing period length which is well simulated by the Markov chain. The average period length is generally shorter than two time steps; only overcast days in winter last, in the average, three days. First passage times (2.8) from one extreme state (e.g. θ_1 = OVC) to another (e.g. θ_4 = CLR) have their frequency maximum between two and five days. The probabilities decrease slowly with almost periodic ups and downs of the same length. This is caused by disturbances passing the station, but it is smoothed out by the Markov chain model. The recurrence times (2.8) to the initial state show the phenomenon of persistence; i.e. the system is very likely

■ **Table 4** Average Markov chain and observed (in brackets) occupation times during passage between extreme states.

■ **Tabelle 4** Mittlere Besetztzeiten der Markov-Kette und der Beobachtungen (in Klammern) während eines mittleren Durchgangs zwischen extremen Zuständen.

Mittlere Besetztzeit während des Durchgangs zum absorbierenden Zustand OVC (1) und CLR (4)
Mean occupation time during passage to the absorbing state OVC (1) and CLR (4)

Summer

OVC (1) — im/in Zustand – state; von/from i:	j: 1	2	3	4	Durchgangszeit first passage time
OVC 1	1.0(1.0)	2.8(2.4)	2.6(2.1)	0.6(0.5)	7.0(6.0)
BKN 2	/	4.1(3.6)	3.0(2.3)	0.7(0.5)	7.8(6.4)
SCT 3	/	3.1(2.5)	4.4(4.0)	0.9(0.8)	8.4(7.3)
CLR 4	/	2.9(2.2)	3.7(2.9)	2.3(2.4)	8.9(7.5)

CLR (4) — im/in Zustand – state; von/from i:	j: 1	2	3	4	Durchgangszeit first passage time
OVC 1	3.8(3.6)	7.9(7.4)	6.3(5.7)	/	18.0(16.7)
BKN 2	2.7(2.4)	8.7(8.2)	6.3(5.8)	/	17.7(16.4)
SCT 3	2.4(1.9)	6.9(5.8)	6.9(6.4)	/	16.2(14.1)
CLR 4	1.6(1.0)	4.6(3.0)	4.2(2.8)	1.0(1.0)	11.4(7.8)

Winter

OVC (1) — im/in Zustand – state; von/from i:	j: 1	2	3	4	Durchgangszeit first passage time
OVC 1	1.0(1.0)	0.4(0.4)	0.3(0.2)	0.1(0.1)	1.8(1.7)
BKN 2	/	1.5(1.5)	0.3(0.4)	0.1(0.1)	1.9(2.0)
SCT 3	/	0.5(0.5)	1.6(1.7)	0.2(0.2)	2.3(2.4)
CLR 4	/	0.6(0.5)	0.6(0.8)	1.5(1.5)	2.7(2.8)

CLR (4) — im/in Zustand – state; von/from i:	j: 1	2	3	4	Durchgangszeit first passage time
OVC 1	26.4(14.7)	9.5(5.6)	6.4(3.7)	/	42.3(24.0)
BKN 2	24.9(12.1)	10.4(6.2)	6.3(3.5)	/	41.6(21.8)
SCT 3	23.2(10.9)	8.8(4.9)	7.1(4.4)	/	39.1(20.2)
CLR 4	17.6(6.8)	6.7(2.8)	4.7(1.9)	1.0(1.0)	30.0(12.5)

to return to the initial state one day later, despite the short mean period length. The Markov chain describes this effect with high accuracy. Averages of period length, first passage time and recurrence time are deduced (Table 3) from the observed and Markov chain probability distributions (2.7, 2.8). Although not presented, it should be mentioned that standard deviations and means are of about the same magnitude. Largest discrepancies between the first order Markov chain and the empirical (in brackets) averages occur for all winter first passage times to the clear state (θ_4 = CLR). The rarely observed clear sky state in winter ($\pi_4 = 0.03$, Table 2), leads to unstable model estimates and explains why model and observed distributions hardly coincide. All other mean values show good to excellent agreement.

A mean occupation time (2.9) defines the average number of model days, on which various states are occupied by the Markov chain while passing to a final or returning to an initial state. As all mean occupation times (Table 4) add up to the related mean first passage or recurrence time, they provide further useful information on the structure of the Markov chain. The basic patterns of the sunshine Markov chain also influence the magnitude of the mean occupation times; the main attractors are defined by persistency and by the most frequently observed states (BKN and SCT in summer, OVC in winter).

■ **Table 5** Hindcast and independent forecast skill of all sunshine states. Brier scores of Markov chain, climate, random and persistence forecasts (B_M, B_c, B_r, B_p); Skill scores of Markov chain predictions compared with climate random and persistence forecasts (S_c, S_r, S_p).

■ **Tabelle 5** Vorhersage-Güte aller Sonnenscheinklassen; Brier scores der Markov-Ketten-, Klima-, Zufall- und Persistenz-Vorhersagen (B_M, B_c, B_r, B_p). Güte der Markov-Kette im Vergleich zur Klima-, Zufall- und Persistenz-Vorhersage (S_c, S_r und S_p).

Hindcast – Independent forecast

Summer: pred. / score	hindcast (25 summers) 1 d	2 d	summer 78 1 d	2 d	summer 79 1 d	2 d	summer 80 1 d	2 d
B_M	.43	.46	.43	.46	.40	.42	.44	.48
B_c	.47	.47	.47	.47	.43	.43	.49	.49
B_r	.56	.56	.56	.56	.56	.56	.56	.56
B_p	.57	.63	.58	.69	.51	.55	.57	.59
S_c	.09	.02	.09	.02	.07	.02	.10	.02
S_r	.23	.18	.23	.18	.29	.25	.21	.14
S_p	.25	.27	.26	.28	.22	.24	.23	.19

Winter: pred. / score	hindcast (25 winters) 1 d	2 d	winter 78/79 1 d	2 d	winter 79/80 1 d	2 d
B_M	.36	.38	.35	.36	.34	.35
B_c	.38	.38	.36	.36	.36	.36
B_r	.56	.56	.56	.56	.56	.56
B_p	.50	.53	.42	.55	.44	.50
S_c	.05	.00	.03	.00	.06	.03
S_r	.36	.32	.38	.36	.39	.38
S_p	.28	.28	.17	.34	.23	.30

(iii) Hindcast and forecast (Table 5): The final decision on a potential applicability of Markov chain predictions is based on the hindcast and independent forecast skill (Section 2.3). The Brier scores, B_M, are evaluated for predictions of the first order sunshine Markov chain as a whole. They are compared with reference models of lead time independent forecast probabilities (B_c: climate; B_r: random; B_p: persistence). The related skill scores, S, are added for convenience to estimate the relative (percentage) improvement of the sunshine Markov chain over its reference models (S_c: climate; S_r: random; S_p: persistence). Table 5 shows the results of one and two day predictions. Hindcasts and independent forecasts are of the same quality and, of course, reveal the tendency of skills decreasing with lead time. The Markov chain beats its most important competitor, the climate model, C_o, by about 10% (5%) in summer (winter). Both climate and Markov forecasts skills can hardly be distinguished at the lead time of two days when the Markov chain loses its memory of the initial state. The skill of other reference models (persistence and random) can be neglected; Markov chains perform about $S_p \sim 25\%$ better than persistence.

(iv) An application: Instead of summarizing the results we apply the above Markov chain analysis to a series of specific user orientated questions, which may arise and for which answers from a stochastic model seem to be reasonable: Say, today has been a sunny or clear (θ_4 = CLR) summer day; i.e. the initial state probabilities are $\theta_i = 0$ für $i = 1, 2, 3$ (OVC, BKN, SCT) and $\theta_4 = 1$ for CLR. Now, the Markov forecaster can be asked about

a) the probability of another clear day tomorrow; the transition probability is $p_{44} \sim 38\%$ (Table 2);
b) the quality of this forecast compared to less competent forecasters as the climatologist or the persistence; the skill scores reveal a $S_c \sim 8\%$ or $S_p \sim 25\%$ improvement (Table 5);
c) the forecaster's expectation of the length (average and distribution) of the sunny period; the average period length is about one to two days (Table 3);
d) the forecaster's expectation of the next clear or overcast day (because the average clear period lasts less than two days); the Markov recurrence and first passage times are more than a week (Table 3);
e) how many broken, scattered or overcast days are expected; between a clear and overcast day the Markov forecaster expects occupation times of 2.9 broken, 3.7 scattered, and 2.3 clear days, and between a clear and another clear day the Markov forecast yields 1.6 overcast, 4.6 broken, and 4.2 scattered days (Table 4).

4 A Berlin Rainfall Markov Chain

The predictand rainfall should not be separated from those weather phenomena to which it is closely related. The synoptic scale dynamics and its vertical motion field must be connected with the rainfall process, if medium range forecasts are considered. For our purposes of stochastic short range predictions, the rainfall will be linked with cloudiness; i.e. dry and wet weather states are predicted by a closed meso-scale information set. This information is contained in the past weather data characterizing ten distinct rain, cloud cover, etc. states, which are mutually exclusive. These weather states are regularly observed at synoptic stations for three or six hourly time intervals, which represent the weather developments by a higher time resolution than the daytime averaged sunshine or cloud cover data (Section 3).

4.1 The observations: past weather states (W_i)

At Berlin-Dahlem three hourly observations of the past weather development are documented in the past weather (W) code (Berliner Wetterkarte, 1970–81). This code (W) contains ten different weather states, which exclude one another at the same time. The ten components of past weather states can be reduced to five (or less), if states rarely observed are combined with meteorologically more relevant ones.

This leads to three states classifying the total cloud cover (in octas):

$i = 1$ (○) : $\leqslant 4/8$
$i = 2$ (◑) : $\sim 4/8$
$i = 3$ (●) : $> 4/8$ or fog;

and two further states describing the rainfall process:

$i = 4$ (●̦) : rain or snow or drizzle
$i = 5$ (▼) : shower or thunderstorm

Sandstorms are not observed. These five classes define the components of a state probability vector

$(W_i) = (W_1, W_2, \ldots) =$ (○, ◑, ●, ●̦, ▼)

which is to be predicted by a Markov chain using a time step of three hours.
Forecast verifications are based on to two alternative models (Section 4.2):
a minimum two state Markov chain of a single wet state ($i = 4$ or 5) and a single dry state ($i = 1$ or 2 or 3):

$(W_{1,2,3}, W_{4,5}) = (W_i)$ and

a four state Markov chain with the same wet state ($i = 4$ or 5) but three dry or cloud cover states ($i = 1,2,3$):

$(W_1, W_2, W_3, W_{4,5}) = (W_i)$.

The Markov chain model is fitted to the weather data of ten summer (June to August) and ten winter seasons (December to February) observed at Berlin-Dahlem from 1970 to 1980, summer 1981 and winter 1980/81 are left for independent forecasts. The beginning and the end of a season is required to coincide with the beginning or the end of a weather state period, which should not be interrupted artificially. This, and the fact that all seasons start independently of each other, lead to small discrepancies in the accumulated joint frequencies of present and past states (Table 6). The following discussion on model identification, estimation and analysis is reduced to a necessary minimum, because the forecasting of rainfall probabilities (W_4, W_5 or $W_{4,5}$) is emphasized in this section.

■ **Table 6** Estimated joint probability distributions and transition probabilities for present (t) and preceeding (t – 1) past weather states (W_i). Bottom: Equilibrium (climate) state occupation probabilities.

■ **Tabelle 6** Geschätzte Wahrscheinlichkeitsverteilungen und Übergangswahrscheinlichkeiten zwischen gegenwärtigen (t) und vorangegangenen (t – 1) Zuständen des vergangenen Wetters (W_i). Untere Reihe: Gleichgewichts-(Klima-) Wahrscheinlichkeiten der Besetzung von Zuständen W_i.

Sommer Summer	Häufigkeiten – frequencies						P_{ij}: Übergangswahrscheinlichkeiten ± 95% Konfidenz transition probabilities ± 95% confidence limts					
t / t – 1	Zustand – state						Zustand – state					
	j: ○	◑	●	●̦	▼	Σ	j: ○	◑	●	●̦	▼	Σ
i: ○	1674	449	4	1	15	2143	.781 ± .018	.209 ± .017	.002 ± .002	.001 ± .001	.007 ± .004	1.
◑	418	859	452	44	125	1898	.220 ± .019	.453 ± .022	.238 ± .019	.023 ± .007	.066 ± .011	1.
●	2	387	1140	225	242	1996	.001 ± 001	.194 ± .017	.571 ± .022	.113 ± .014	.121 ± .014	1.
●̦	6	50	212	316	74	658	.009 ± .020	.076 ± .020	.322 ± .036	.480 ± .038	.113 ± .024	1.
▼	39	157	189	72	198	655	.059 ± .018	.240 ± .033	.289 ± .035	.110 ± .024	.302 ± .035	1.
Klima climate	2139	1902	1997	658	654	7350	.291 ± .010	.259 ± .010	.272 ± .010	.089 ± .006	.089 ± .006	1.

Joint frequencies are derived and normalized to obtain the maximum likelihood estimates of the past weather transition probabilities between three hourly time intervals. Again, first order Markov chains describe the data set significantly better than the zero order Markov chains, which are defined by the equilibrium state occupation probabilities, i.e. there are predictable Markov properties in the past weather time series. Higher order models hardly improve the hindcast skill. If both shower and rainfall states are combined, the Markov chains reveal seasonally stable structures; i.e. seasonal variations of transition probabilities occur within the 95%-confidence intervals. Therefore, only the transition probabilities of the summer Markov chain are documented (Table 6). The model diagnosis shows that, in general, the Markov chain distributions do not significantly deviate from the empirical distributions (applying chi-squared tests with 95% significance levels). The related averages of model and observed first passage times, recurrence times and period lengths are useful forecast aids and therefore documented in Table 7.

■ **Table 7** Average first passage times and period lengths of the past weather Markov chain and of the observations (in brackets). The diagonal values are mean recurrence times π_j^{-1}.

■ **Tabelle 7** Mittlere Durchgangszeiten und Periodenlängen der Wetter-Markov-Kette und der Beobachtungen (in Klammern). Die Diagonalelemente sind mittlere Rückkehrzeiten π_j^{-1}.

Summer	Mittlere Durchgangszeit in Stunden mean first passage time in hours					mittlere Perioden mean periods
nach / to von / from	Zustand – state j: ○	◑	●	,	▼	L
i: W_1 = ○	10.3(10.1)	14.4(25.4)	35.7(48.2)	82.8(115.3)	60.5(83.4)	13.7(13.6)
W_2 = ◑	33.1(31.7)	11.6(11.5)	22.5(22.7)	69.9 (96.5)	48.4(58.8)	5.5 (5.5)
W_3 = ●	46.9(50.4)	16.6(19.8)	11.0(10.9)	55.3 (68.9)	39.0(50.3)	7.0 (7.0)
W_4 = ,	49.0(57.0)	19.6(25.1)	13.4(12.7)	33.4 (31.5)	38.1(44.9)	5.8 (5.8)
W_5 = ▼	42.8(46.3)	15.5(17.7)	17.2(15.5)	58.2 (66.2)	33.6(32.8)	4.3 (4.3)

Winter	Mittlere Durchgangszeit in Stunden mean first passage time in hours					mittlere Perioden mean periods
nach / to von / from	Zustand – state j: ○	◑	●	,	▼	L
i: W_1 = ○	20.0(19.5)	14.5(29.1)	26.0(27.0)	39.0(52.3)	120.0(227.4)	11.5(11.2)
W_2 = ◑	62.9(56.3)	19.3(18.8)	15.5(13.4)	29.0(36.8)	110.1(162.4)	4.8 (4.8)
W_3 = ●	87.9(83.2)	31.8(40.1)	7.5 (7.5)	19.7(26.9)	107.1(158.3)	8.9 (8.9)
W_4 = ,	88.5(90.6)	33.9(48.1)	12.2(13.1)	11.4(11.1)	104.7(133.9)	7.8 (7.7)
W_5 = ▼	78.0(86.8)	27.5(30.3)	14.0(14.9)	21.6(18.4)	83.4 (68.6)	4.1 (4.1)

In the following it should be realized that a past weather state characterizes the weather development during the preceeding three hours; i.e. the past weather set as a whole evolves continuously, although it is forecasted in discrete 3h time steps. This is a time analogue of layer versus level models.

4.2 Forecasting rainfall probabilities

The estimated transition matrices (Table 6) define the Markov chain models by which probabilities of rainfall and other past weather states can be predicted. Several different Markov chain forecasts are made and compared to demonstrate a few potential applications. They start either from the dry states of a low or high amount of cloud cover, or from a single wet state ($W_{4,5}$) combining rainfall and showers (i = 4 and 5). Accordingly, forecasts are performed by a Markov chain of five, or four states, respectively. Both point (time step) and integral (time interval) forecasts are discussed, and the diurnal cycle modifying the summer forecasts.

(i) Time step and time interval predictions (Figure 4): The common lead time n predictions are obtained by the n-th power of the Markov chain transition matrix, $(p_{ij})^{(n)}$. This leads to time trajectories of past weather state probabilities, $(W_i)^{(n)}$, which depend on the forecast time step n (see Section 2.1). The model behaviour is displayed in Figure 4. The time trajectories start from two different initial conditions at $n = 0$ with state probabilities zero or one, respectively; if there is low cloud amount at $n = 0$, then $W_1 = 1$ and $W_i = 0$ for $i = 2,\dots,5$; a wet state at $n = 0$ means $W_{4,5} = 1$ and $W_i = 0$ for $i = 1,2,3$. As the time evolves, all state probability trajectories, $(W_i)^{(n)}$, tend towards their equilibria, (π_i), by which the climate state or reference prediction is defined. A related e-folding time scale τ (2.6) is derived from the eigenvalue λ_1 (Section 2.1). While approaching the equilibrium, the Markov chain reduces its memory of the initial condition by 1/e after about four time steps; in summer $\tau \sim 12$ h, in winter $\tau \sim 13$ h. The equilibria, which are independent of the initial conditions, are indicated at the ordinate (i.e. state probability axis).

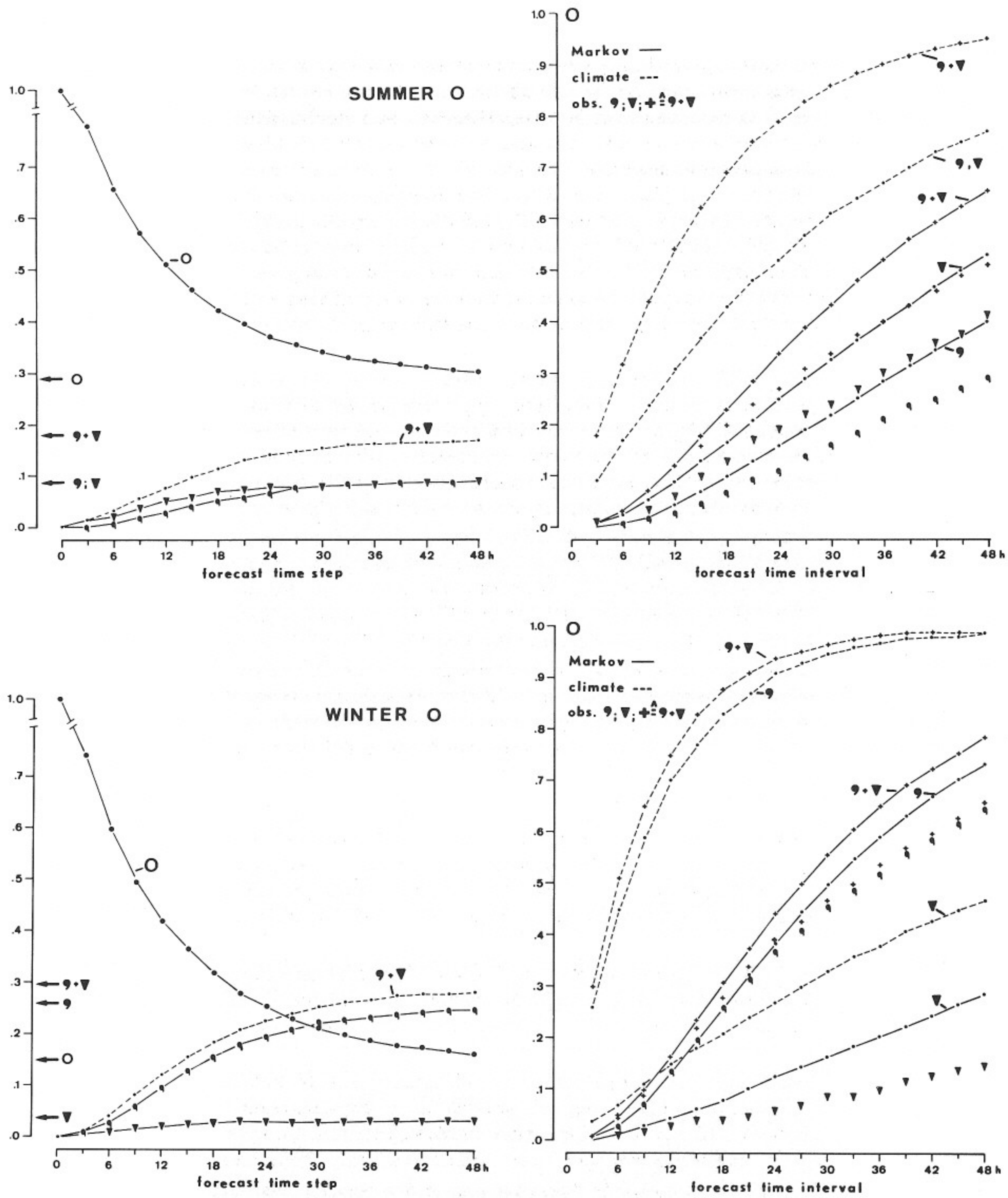
The point forecasts are complemented by probability predictions of first passage and recurrence times, $(f_{ij})^{(n)}$, integrated (or accumulated) over an interval of consecutive time steps. Such a forecast scheme defines probability measures, which quantify the chances of rainfall and shower occurring (at least once) within a specified time interval (Sections 2.1 and 2.3). With increasing length of the forecast time interval the first passage or recurrence time probabilities accumulate to one. As all states form a closed set they can pass from one to another or return to the initial state at least once during an infinitely long time interval.

Such integral (or interval) forecasts can be made by Markov chain models (full lines in Figure 4). The results are compared with observations (dotted symbols) and with predictions by the related zero order climate or equilibrium probability matrix, C_0 (dashed).

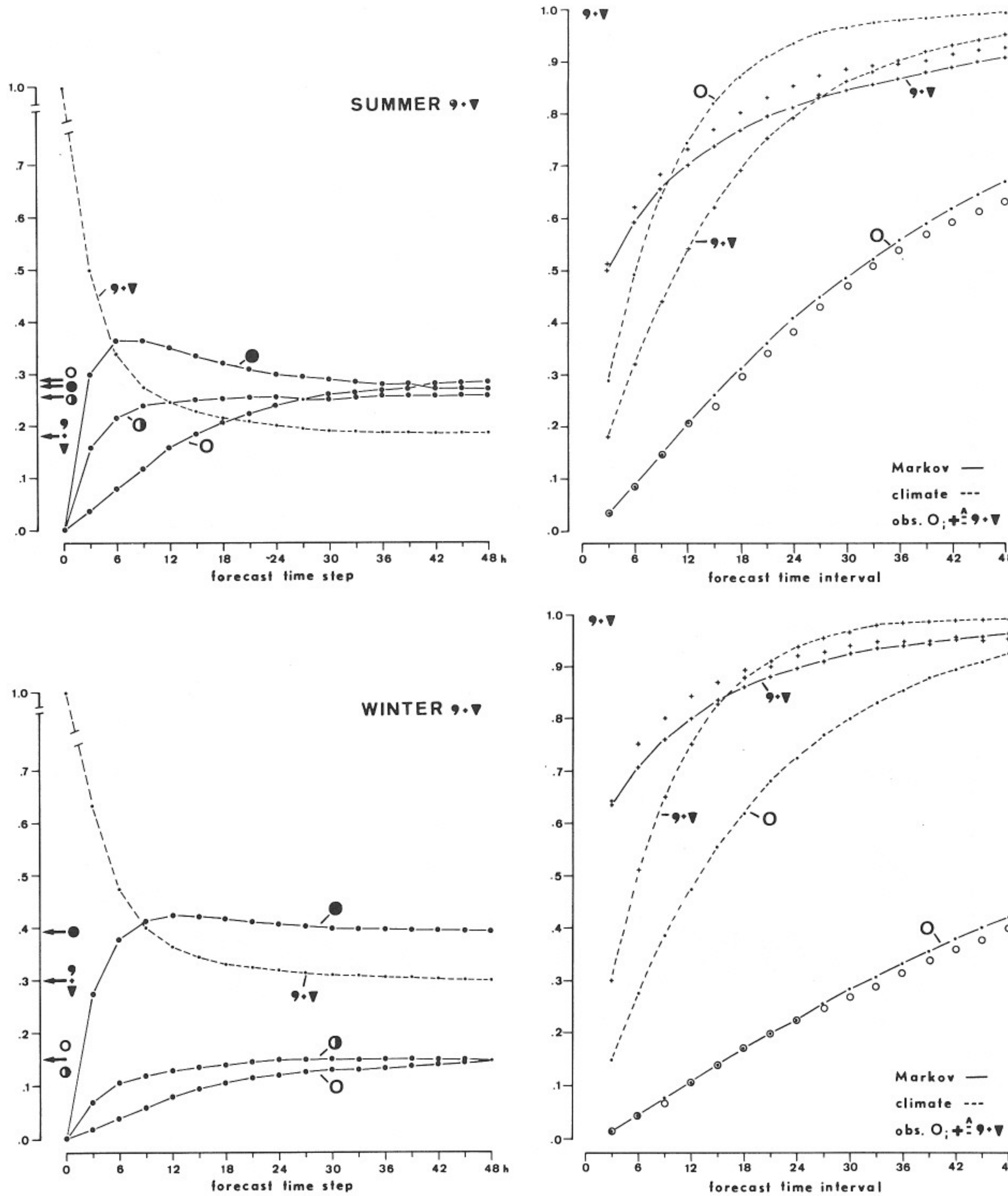
There are significant deviations between the first and zero order Markov chains. Only the first order chain fits the observations within a lead time interval up to 24–36 h. The Kolmogoroff-Smirnov test has to be applied to these accumulated distribution.

(ii) Forecast skill (Figure 5): Both the time step and the time interval probability forecasts are quantitatively evaluated for a single wet state ($W_{4,5}$), which combines rainfall (W_4) and shower (W_5). Thus, wet state Brier scores, B_{M4} (dashed line) and skill scores, S_{M4} (full line) are determined by four state Markov chain predictions. Additionally, this wet state prognosis is compared with analoguous results from an alternative Markov chain which has been reduced to a minimum of two states, wet and dry (Section 4.1).

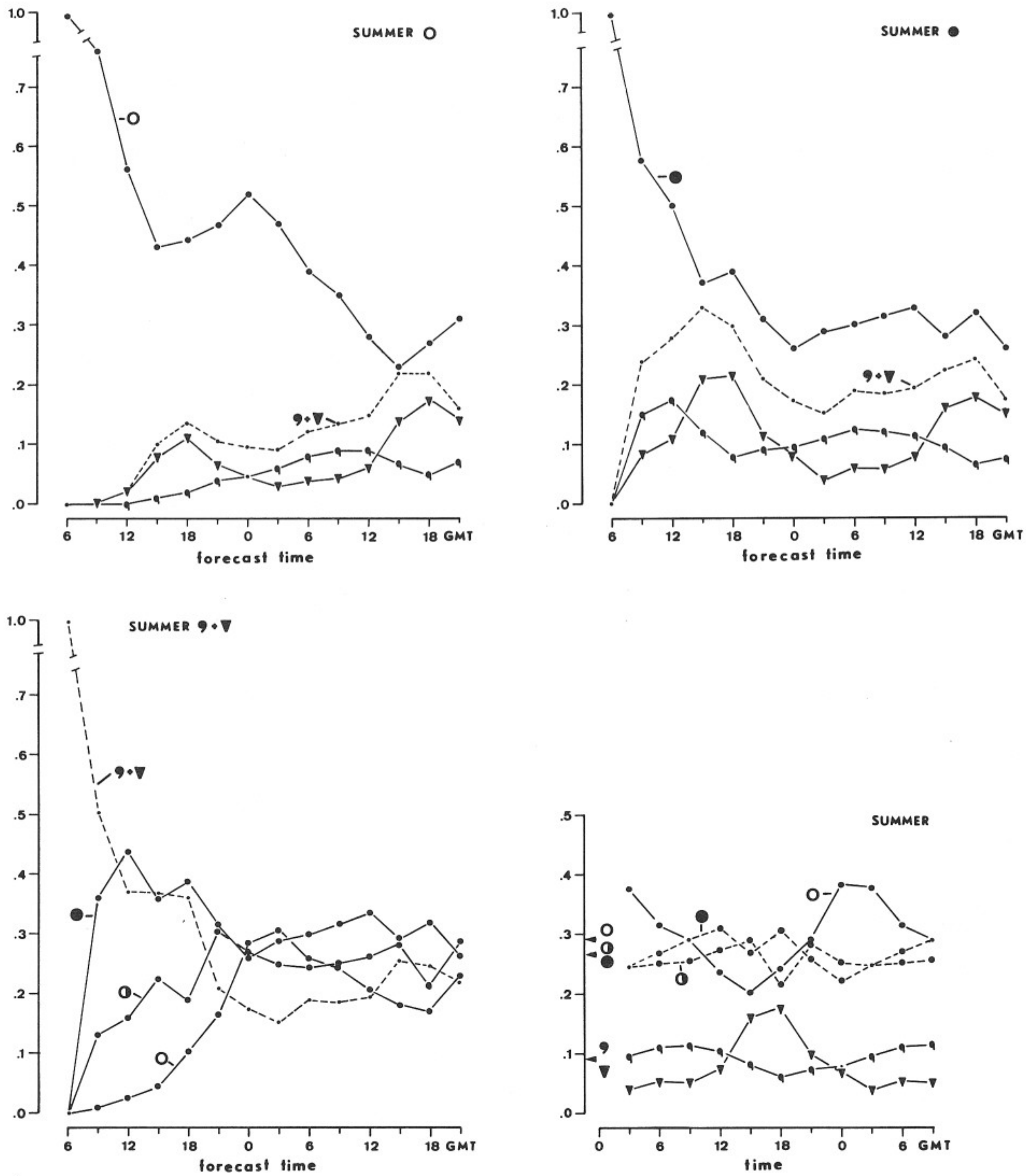
The initially high skill score of the Markov chain point forecasts decreases with increasing lead time. After about one day (i.e. 2τ or 8 time steps) the Markov chain Brier score, B_M, can hardly be distinguished from the reference forecasts, B_c, by the climate state probabilities (i.e. $S_c \sim 0$). Although the summer model predicts the wet state probabilities better than the winter Markov chain (B_{M4} (summer) $>$ B_{M4} (winter)), the improvement over the reference model forecasts is almost the same for both seasons. The



● **Figure 4** Summer and winter forecast probabilities of wet weather states (rainfall and/or showers) for increasing time steps (left); accumulated probabilities of wet state passage or return times within time intervals (right). The forecast probabilities depend on the initial conditions which are indicated at the top. Markov chain forecasts (full line) are compared with observed (dotted) and climate state probabilities (dashed).

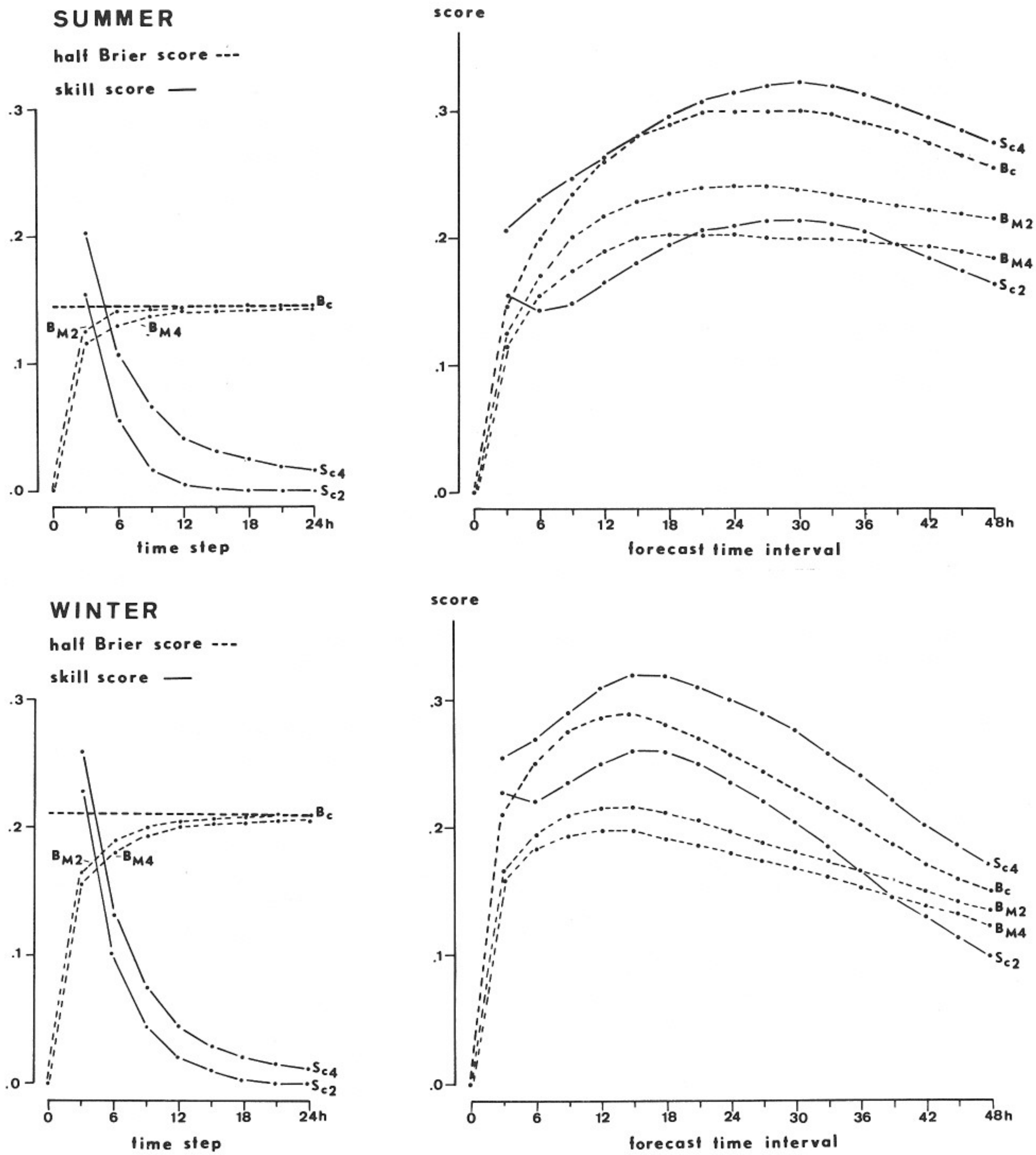


● **Bild 4** Jahreszeitliche Vorhersagewahrscheinlichkeiten der nassen Wetterzustände (Regen und/oder Schauer) in Abhängigkeit vom Zeit-Schritt (links); akkumulierte Wahrscheinlichkeiten für Durchgangs- oder Rückkehrzeiten der nassen Wetterzustände innerhalb von Zeitintervallen (rechts). Die Vorhersagewahrscheinlichkeiten hängen von den Anfangszuständen ab, die oben angegeben sind. Vorhersagen mit Markov-Ketten (durchgezogen) werden verglichen mit den beobachteten (Punkte) und den Klima-Wahrscheinlichkeiten (gestrichelt).



● **Figure 5** Summer forecast probabilities and the diurnal cycle. All forecasts start at 6 GMT where initial states attain probability one.

● **Bild 5** Wahrscheinlichkeitsvorhersage und Tagesgang im Sommer. Die Prognosen beginnen um 6 Uhr GMT, wobei Anfangszustände von der Wahrscheinlichkeit eins ausgehen.



● **Figure 6** Hindcast verification scores of time step (left) and time interval (right) wet state probability prediction; the half Brier score, B, and the skill score, S, are defined by forecasts of four and two state Markov chains (B_{M4}, B_{M2}, S_{C4}, S_{C2} and climate probabilities (B_C).

● **Bild 6** Güte-Maße von Zeit-Schritt (links) und Zeit-Intervall (rechts) Vorhersagen der Wahrscheinlichkeit eines nassen Wetterzustandes: Brier- (B) und Skill-score (S) ergeben sich aus Prognosen mit Hilfe der Markov-Ketten mit vier und zwei Zuständen (B_{M4}, B_{M2}, S_{C4}, S_{C2}) und der Klima-Wahrscheinlichkeiten (B_C).

minimum Markov chain provides comparatively poor probability prediction scores (B_{M2}, S_{c2}) in summer. But, as almost 80% of all winter days have overcast or broken sky conditions (Section 3.1), the additional prediction of cloud cover states in winter does not considerably improve the wet state forecasts.

Within short and long time intervals the Markov chains perform well in predicting first passage or return times to the wet state. Although time intervals of about one day are relatively poorly predicted (according to the high Brier scores B_M), forecasts show maximum skill score, S_c, if compared with its climatic reference. Finally, it should be noted that both the independent forecast and hindcast scores are of the same magnitude.

(iii) Diurnal cycle (Figure 6): In summer the diurnal cycle modifies the rainfall considerably. If this effect is added to a probability prediction scheme, it can improve practical forecasts. An example is briefly discussed in the following. Three hourly transition probability matrices are evaluated at fixed hours (GMT) of the day and the related probability forecasts are made accordingly; after each time step the transition matrices are replaced by their successors. The resulting past weather probability trajectories, which include the diurnal cycle, are shown in Figure 6. They start at 6 GMT from various initial conditions ($W_1 = 1$, $W_3 = 1$ and $W_{4,5} = 1$).

Obviously, all trajectories are attracted by the diurnal "limit" cycle, which is defined by the climate or equilibrium state occupation probabilities at the fixed hours of the day. It should be mentioned that the cycle average is identical with the overall equilibrium state probabilities (π_i) of the Markov chain (Table 6), which is estimated by disregarding diurnal effects. Again, after about one day the initial conditions are lost by the forecast model and the diurnal limit cycle dominates the forecast probabilities.

■ **Table 8** 12 h-forecasts of the summer rainfall and shower probabilities, conditional on various initial past weather states W_i. From left to right: time step and time interval forecasts independent of the diurnal cycle; forecasts from 6 to 18 GMT with diurnal cycle (see text).

■ **Tabelle 8** 12-Stunden-Vorhersage sommerlicher Niederschlags- und Schauerwahrscheinlichkeiten in Abhängigkeit vom Anfangszustand des Wetters. Von links nach rechts: Zeit–Schritt- und Zeit–Intervall-Vorhersage unabhängig vom Tagesgang; Vorhersagen von 6 nach 12 GMT unter Berücksichtigung des Tagesgangs (siehe Text).

Summer		point forecast			interval forecast			diurnal cycle		
12 h forecasting		state probabilities after 4 time steps			passage or recurrence probabilities in 12 h time interval			state probabilities at 18 GMT		
from state	to state	Markov chain	obs.	climate	Markov chain	obs.	climate	Markov chain	obs.	climate
○	❟	.03	.02	.09	.04	.05	.31	.02	.02	.06
	▼	.05	.05	.09	.03	.10	.31	.12	.11	.18
	❟ + ▼	.08	.07	.18	.12	.12	.54	.14	.13	.24
●	❟	.13	.13	.09	.31	.30	.31	.09	.05	.06
	▼	.12	.10	.09	.36	.34	.31	.21	.23	.17
	❟ + ▼	.25	.23	.18	.57	.53	.55	.30	.28	.23
❟ + ▼	❟ + ▼	.25	.30	.18	.70	.73	.55	.31	.38	.23

(iv) An application (Table 8): A specific example is presented for 12-hour predictions of rainfall and shower probabilities in summer. The forecast time length corresponds to the e-folding time scale τ. The probability predictions start from various initial past weather states (W_i) and are determined by the fitted (five state) Markov chain model (Table 6). Point forecasts are evaluated to yield probabilities at the fourth time step which cover the past weather between 9–12 h; interval forecasts provide the wet state first passage (or return) probability within the time interval from 0 to 12 hours. Finally, the diurnal cycle is also considered, for which the initial conditions are fixed at 6 GMT; the wet state probabilities are predicted for 12h in advance. i.e. the 18 GMT prognosis covers the time between 15 to 18 GTM. The results are compared with the climate or reference forecast model and observations. The latter are joint probabilities estimated from observed 12 h time steps probabilites and from the observed first passage distributions. The Markov chain predictions and empirical probabilities are in good agreement, from which the climate forecasts, however, deviate.

State probabilities of shower and rainfall are additive, if point forecasts are considered. They are valid for a past weather interval of 3 h ending at the forecast time; e.g. a twelve hour prediction means the past weather from 9 to 12 h. Forecasting 12 h in advance yields 8% or less chances of rainfall and showers, if the initial state has low amount of cloud cover (W_1). The chances increase to 14%, if afternoon forecasts are made, and the diurnal cycle is included. The probabilities rise even further to 25% without (and 31% with) the diurnal cycle, if initial past weather states tend towards overcast or wet weather conditions.

Considering the wet state interval forecasts (i.e. first passage probability accumulations), shower and rainfall are, in general, not additive. If the cloud amount is low, there are 12% chances of rain and/or shower occurring at least once within the next twelve hours. If rainfall and/or showers have occurred at the beginning of the forecast interval, the chances rise to 70%, which is due to persistence. The probability of rain and/or shower within the next 12 h is 57%, when the cloud amount has been high, initially.

Additional forecast probabilities may be extracted from these prediction schemes. But, our purpose was to present some simple models and evaluate examples of probability forecasting at a single station. Furthermore, this study may help to emphasize the relevance of this type of probability prediction (see e.g. MURPHY and WILLIAMSON, 1976), and possibly contributes further aspects to it.

5 Outlook

Probability forecasting by elementary stochastic models is applied to short range weather prediction using single station observations only. In this pilot study we have derived and applied probability prediction schemes to support short range forecasts or nowcasts issued by local weather services. The capacity of these models is, of course, not infinite, but for a single station forecaster some advantages seem to be evident.

(i) The models are fitted to the local climatology of the station for which the forecaster has to formulate the prediction. Thus, the model transfers local weather history by a linear Markov chain memory into the future. Furthermore, the inherent randomness or uncertainty of the weather process is quantitatively included in the forecast.

(ii) Stochastic single station models can be run any time when a surface observation becomes available. They do not depend on the fixed upper air cycle, no initialization is required, and almost no computer time is consumed. Therefore, these models may help to close a time lag of 6–12 h, before the latest deterministic nonlinear numerical weather predictions, based on the upper air cycle, are available.

(iii) Finally, linear stochastic forecast models are transparent and simple. As they provide the weather information adequately in probabilistic terms, these models seem to be a useful local forecast guidance. However, comprehensive mesoscale models nested into a numerical weather prediction scheme are a completely different approach to guide local forecasters.

Of course, numerical weather prediction models, which include model output statistics, are superior to this classical stochastic approach after, say, 24 h (KLEIN, 1982). This gap, however, can be filled by stochastic models, which may develop in the future.

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