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# Improved tropical cyclone track predictions using error recycling

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With 2 Figures

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**Summary**

Errors produced by a nonlinear predictive scheme contain information about both the observations and the prediction system. Therefore, its error history would be expected to contribute to increasing the skill of the predictions if it is included in the forecast. In this study an error recycling procedure is developed for tropical cyclone track prediction. Errors are defined here as differences between the model forecast and the best track position. Error histories are incorporated into a nonlinear analogue, or simplex, forecast scheme and applied to tropical cyclone track prediction, using the archives of observed position data associated with the forecast errors. Various forecast experiments of the cyclone tracks are performed: standard simplex predictions using observed positions only; simplex predictions improved by error forecasts based on libraries of both observations and the recycled forecast errors; and, finally, predictions that include NWP-model forecasts and their errors as predictors. The resulting gains in skill of predictions out to 72 hours ahead are found to be substantial.

## 1. Introduction

One of the most important applications of weather forecasting is the prediction of tropical cyclone motion. Meteorological services in tropical regions affected by tropical cyclones regard the prediction of these systems as their major weather forecasting problem. Tropical cyclones, also known as hurricanes and typhoons in different parts of the world, are the most destructive of all natural hazards in terms of loss of life and property. Although tropical cyclone position forecasting has been a major activity for many decades, it has defined rapid improvement, with an average of about one per cent reduction per annum in mean 48 hour position errors over the past decade or so. It is noted that there has long been two distinct approaches to tropical cyclone position forecasting. One kind is based on statistical regression schemes, like CLIPER, which as the name suggests is a blend of climatology and persistence forecasts. Complete descriptions of the approach may be found in Neumann (1972), Leslie et al. (1990) and many other sources. The second and major approach employs deterministic algorithms, the main example being numerical weather prediction (NWP) models.

Traditionally, the statistical and deterministic methods are used separately although there is some work in which the techniques have been linearly combined in a least squares error minimizing fashion (see, for example, Thompson, 1977; Fraedrich and Leslie, 1987). Such optimal weighting of forecasts has also received much attention in the economics, management and statistics literature. In meteorology it is also known that consensus forecasts provide in the average more accurate results than the individual forecasts which comprise the consensus.

Although this is an incontrovertible fact, it still does not appear to be widely recognized or accepted. The linear error minimizing multivariate combination forecast of tropical cyclone positions shows considerable forecast improvement, especially when the numerical weather prediction (NWP) model is combined with an empirical forecast system based on nonlinear analogue or statistical regression techniques (for example, CLIPER). Independent forecast trials by the authors have shown that the 24, 48 and 72-hour position error of the combination forecast can be reduced by 15–20%. Skill improvement of that magnitude normally requires considerable investment in the development of predictive schemes, whereas the combined dynamical-statistical schemes described above produce gains at almost zero cost. For additional details, the reader is referred to Fraedrich and Leslie (1987), and Leslie and Fraedrich (1990). Information on the practical implementation of the scheme at the Joint Typhoon Warning Center (Guam) and on the education of new typhoon duty officers are given by Mundell and Rupp (1995).

In this study we present another example of our thesis of "obtaining significant increases in skill for almost insignificant cost", in the short term forecasting arena. As mentioned above, this approach was applied with success over a decade ago, beginning with the study by Fraedrich and Leslie (1987). Here, we utilize the error history of prediction schemes usually considered as forecast dross. Errors occur in all practical prediction models and traditionally are used in two main ways: most commonly as a measure of the skill of the model; or, occasionally, to improve predictions by, for example, regressing errors against observations. An entirely different approach is presented here, together with an application to forecasting tropical cyclone tracks. The methodology is described in Sect. 2, results are presented in Sect. 3, and conclusions and discussion comprise Sect. 4.

## 2. Nonlinear forecasts with recycled errors

The standard approach to prediction can be referred to as *dualistic*. That is, two separate dynamical systems are formed, one by the prediction vectors, $\mathbf{X}^*$, generated by an algorithm applied to past and present observations, and the other by the corresponding verification data set, $\mathbf{X}$. The forecast error vectors, defined as $\mathbf{e} = \mathbf{X}^* - \mathbf{X}$, provide the link between the two. However, forecast errors commonly are not utilized as they are thought of as white noise, which does not contain any useful information about either $\mathbf{X}^*$ or $\mathbf{X}$. Relaxing this assumption leads to the *monistic* approach: forecasts and verifying observations are treated as a unified dynamical system characterized by the expanded state vector $(\mathbf{X}, \mathbf{e})$ rather than the original state vector $\mathbf{X}$ and the corresponding predicted state vector $(\mathbf{X}^*, \mathbf{e}^*)$, instead of simply $\mathbf{X}^*$. Thus, if the errors contain information about the observed and the prediction system, the error history should be expected to contribute to improving the forecasts.

A nonlinear predictive scheme may simply be written as $\mathbf{X}^* = \mathbf{S}(\mathbf{X})$, where $\mathbf{S}$ is in general a nonlinear forecasting algorithm, $\mathbf{X}^*$ is the predictand at $t^* + \tau$ with lead time $\tau$. $\mathbf{X}(t)$ are the verifying observables, wich also enter the forecast algorithm, $\mathbf{S}$, for $t < t^*$. The forecast algorithm adopted here is the widely-used (nonlinear) simplex method of Sugihara and May (1990) and is just one of a range of models that might have been chosen. It is a form of analogue prediction which, for a time-delay embedding of dimension $m$, utilizes the $m+1$ nearest neighbors in the time-delay coordinate phase-space chosen from the "library" or history of the univariate observations. That is, the predictor phase space is spanned by the $m$ time-delay components of $\mathbf{X} = \{\mathbf{X}(t), \mathbf{X}(t-\tau), \mathbf{X}(t-2\tau) \ldots \mathbf{X}(t-(m-1)-\tau)\}$ with sampling time $\tau$. The parameter $m$ is chosen to minimize the forecast error at the first time-step, $t+\tau$, so that future mappings may be performed. The time evolution of the centroid of the enclosing simplex provides the forecast. Other techniques, such as the radial basis function approach of Casdagli (1989), and Abarbanel et al. (1993), or other approaches such as neural nets are not considered in this study. Predictions, $\mathbf{X}_1 = \mathbf{S}(\mathbf{X})$, based on the calibration data set, $\mathbf{X}$, provide a first generation, $i = 1$, of the forecast error history, $\mathbf{e}_1 = \mathbf{X}_1^* - \mathbf{X}$. They are recycled into the prediction scheme and enhance the predictor

**Table 1.** Iteration procedure to obtain the library of recycled errors

| **Iteration** | **first** | **second** | ...... | **last** |
|---|---|---|---|---|
| Forecast pair $(\mathbf{X}^*, \mathbf{e}*)$ | $\mathbf{X}_1^* = \mathbf{S}(\mathbf{X})$ | $(\mathbf{X}_2^*, \mathbf{e}_2^*) = \mathbf{S}(\mathbf{X}, \mathbf{e}_1)$ | | $(\mathbf{X}_n^*, \mathbf{e}_n^*) = \mathbf{S}(\mathbf{X}, \mathbf{e}_{n-1})$ |
| Error $\mathbf{e}_i$ | $\mathbf{e}_1 = \mathbf{X}_1^* - \mathbf{X}$ | $\mathbf{e}_2 = \mathbf{X}_2^* - \mathbf{X}$ | | $\mathbf{e}_n = \mathbf{e}_s = \mathbf{X}_2^* - \mathbf{X} \sim \mathbf{e}_{n+1}$ |

phase space from $\mathbf{X}$ to $(\mathbf{X}, \mathbf{e}_1)$ entering the forecast algorithm at $t < t^*$. A second iteration, $i = 2$, is performed with the forecast scheme, $\mathbf{X}_1^* = \mathbf{S}(\mathbf{X}, \mathbf{e}_1)$. It provides a second generation of errors, $\mathbf{e}_2 = \mathbf{X}_2^* - \mathbf{X}$, wich, in general are smaller than the first generation in a mean square sense. Replacing the first error generation by the second one and applying the predictive scheme yields an iteration procedure (see Table 1). The process is continued until error saturation has been established, without further reduction achievable in the ensemble averaged sense, $\langle \mathbf{e}_s^2 \rangle = \langle \mathbf{e}_n^2 \rangle\rangle \sim \langle (\mathbf{e}_{n-1})^2 \rangle$. This saturation error history, $\mathbf{e}_s$, characterizes both the predictive scheme used and the observed system to be predicted. Now, both the forecast variable and the forecast error can be predicted for independent data, $(\mathbf{X}^*, \mathbf{e}^*) = \mathbf{S}(\mathbf{X}, \mathbf{e}_s)$, using the data library, which includes the recycled errors. In the final step, this pair is used to further improve the forecast by error correction, $\mathbf{X}^{**} = \mathbf{X}^* + \mathbf{e}^*$. Typically, for the geophysical systems we have worked with thus far, only three or four iterations are required to achieve saturation, that is, $i \sim 3$ to 4).

## 3. Application to reducing tropical cyclone track errors

We now turn to the multivariate prediction of cyclone positions. First, the predictands, $\mathbf{X}^* = \mathbf{S}(\mathbf{X})$, are taken from the observations of the actual positions, $\mathbf{X}$. Then a library of position errors of past forecasts, $\mathbf{e}$, is established to unify the prediction and verification system, as described by the iteration procedure in Sect. 2. This leads to the prediction algorithm for the forecast and forecast error pair, $(\mathbf{X}^*, \mathbf{e}_s^*) = \mathbf{S}(\mathbf{X}, \mathbf{e}_s)$.

### *3.1 Data set*

The Australian region tropical cyclones and hurricanes are selected for the forecast experiments. The cyclone track data are available for the period 1959 to 1998 at 6 hourly time steps. The library of tracks and of saturation errors established through error recycling is based on the even-years of the data set. The optimal error minimizing embedding dimension of the time-delay coordinates is found to be $m \sim 4$. The odd years chosen for the independent forecasts comprise more than 150 tropical cyclones with an average life-time of about 6 days. The CLIPER statistical regression scheme of Leslie et al. (1990) serves as a reference forecast. All forecast errors are presented in a diagram of rms-distances versus lead-time up to 72 hours.

### *3.2 Simplex forecasts with and without error recycling*

Three sets of simplex experiments are performed: first are standard simplex predictions of observed positions only, $\mathbf{X}^* = \mathbf{S}(\mathbf{X})$; second are simplex predictions of observations and errors using the history of both observed positions and the position errors, $(\mathbf{X}^*, \mathbf{e}_s^*) = \mathbf{S}(\mathbf{X}, \mathbf{e}_s)$, and the last employs the predicted error correction, $\mathbf{X}^{**} = \mathbf{X}^* + \mathbf{e}_s^*$. The following results are shown in Fig. 1. Note that the performance of the standard simplex forecast, $\mathbf{X}^* = \mathbf{S}(\mathbf{X})$, is comparable to the statistical forecast scheme CLIPER in the first 24 hours and improved at longer lead times (denoted by $S$ and $C$, respectively, in Fig. 1). Including the saturation prediction error history in the forecast, $\mathbf{X}^* = \mathbf{S}(\mathbf{X}, \mathbf{e}_s)$, reduces the mean forecast error by about 15 to 20km. This scheme is denoted by SE. Finally, when the forecast $\mathbf{X}^*$ is corrected by the predicted error, $\mathbf{e}^*$, using the simplex forecast pair $(\mathbf{X}^*, \mathbf{e}^*) = \mathbf{S}(\mathbf{X}, \mathbf{e}_s)$, we obtain the best forecast of all, $\mathbf{X}^{**} = \mathbf{X}^* + \mathbf{e}^*$, denoted by SPE and gaining another 10 to 15 km compared with the standard complex scheme, $S$. Expressed in relative terms, the error reduction, relative to CLIPER is about 30% for the SPE scheme, which is very sizable indeed. This relative gain in

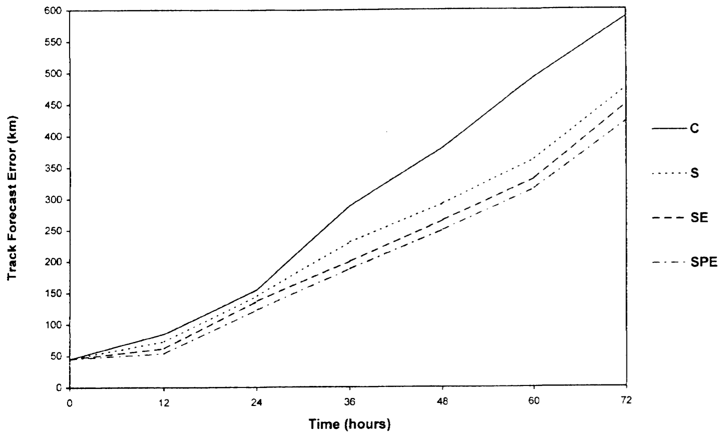


**Fig. 1.** Mean position errors (km) of tropical cyclone track forecasts (in the Australian basin) for up to 72 hours ahead. In addition to CLIPER (C) there are three predictions: standard simplex (S), simplex with error history (SE), and simplex plus error corrections (SPE)

skill is less for the *S* and SE schemes but is still very significant at about 20% and 25%, respectively. The rms error growth in all cases is almost linear in the first two days but increases at a slightly faster rate thereafter. This quasi-linear behavior of the increase in mean tropical cyclone position out to 72 hours is noted elsewhere in this Special Issue, in the article by Leslie and Abbey. It is also worth repeating that the scheme is extremely quick, taking an insignificant amount of time on any machine from a fast PC to a high performance computer.

### *3.3 Nonlinear combination with NWP-forecasts*

A further gain can be achieved by introducing an independent forecast scheme, in this case the numerical weather prediction (NWP) model forecasts (Leslie and LeMarshall, 1998), denoted by the stae vector, $\mathbf{N}$, and their corresponding forecast error histories, $\mathbf{e_N}$. In this sense the simplex prediction scheme operates as a method which combines independent forecasts, namely NWP and standard simplex, in a nonlinear fashion. As before, forecast iterations establish the saturation error history using a test data set and the same iteration procedure described above. The evaluation of the prediction scheme is based on the same independent data set. Figure 2 includes the following results. The standard simplex scheme $\mathbf{S}(\mathbf{X})$, again denoted by *S*; the NWP forecast, denoted by *N*; the standard simplex forecasts plus the NWP prediction, $\mathbf{X}^* = \mathbf{S}(\mathbf{X}, \mathbf{N})$, denoted by SN; and, finally, the forecasts comprising the NWP-model and the simplex error history, $(\mathbf{X}^*, \mathbf{e}_s^*) = \mathbf{S}(\mathbf{X}, \mathbf{e}_s, \mathbf{N}, \mathbf{e_N})$. This forecast is obtained by employing the correction from the predicted error, $\mathbf{X}^{**} = \mathbf{X}^* + \mathbf{e}_s^*$, and is denoted by SNE. The following features are of interest. The NWP forecasts, *N*, are superior to the standard simplex, S, which deteriorates at a larger rate after two days. Note, however, that NWP still has difficulty beating this empiricl method in the first 24 hours. After 24 hours, there is considerable forecast improvement in N of about 15–20 km ove the NWP-model by the nonlinear hybrid schemes, SN and SNE where both the observations and the independent NWP predictions enter into the simplex forecast algorithm. This is not unexpected as the product may be interpreted as a non-linear combination of the standard simplex, S, and the NWP forecasts, that is, $\mathbf{X}^* = \mathbf{S}(\mathbf{X}, \mathbf{N})$. This gain in performance is further increased when not only NWP-model forecasts (and their latest forecast errors) are introduced to the simplex predictor set, but also the simplex errors are utilized after recycling them (as described above), $\mathbf{X}^* = \mathbf{S}(\mathbf{X}, \mathbf{e}_s, \mathbf{N}, \mathbf{e_N})$. Finally, correcting this forecast for the predicted forecast error, $\mathbf{X}^{**} = \mathbf{X}^* + \mathbf{e}_s^*$, leads to an additional large gain. It amounts to of another 15–20 km throughout the forecast interval ranging from 48 to 72 hours. This is a total of about 40 km

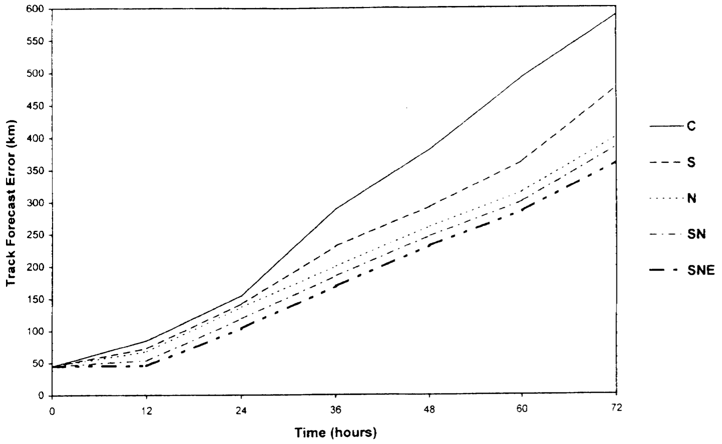


**Fig. 2.** Mean position errors (km) of tropical cyclone track forecasts (in the Australian basin) by four prediction schemes: the standard simplex scheme (S), the NWP model (N), the extended simplex model including the library of the observations and the NWP forecasts (SN) and, finally, the simplex plus NWP error corrections (SNE)

reduction in the 48 to 72 hour forecast range at a negligible computational cost. It also is an improvement over CLIPER of about 35% and relative the NWP model predictions, **N**, it is a sizable 10%.

## 4. Discussion and conclusions

While the conventional approach to forecasting might be termed dualistic, because it treats predictions, $\mathbf{X}^*$, and observations, $\mathbf{X}$, as two totally different entities, the monistic approach, unifies predictions and observations (or forecast errors) into one system $(\mathbf{X}, \mathbf{e})$. In this sense prediction errors, if included in a forecast model, enhance the state space by combining observations and predictions to yield an enlarged state vector. Therefore, predictions of observables and predictions of the forecast error are naturally coupled, $(\mathbf{X}^*, \mathbf{e}^*)$, so that, in a final stage, the forecasts can be improved by the predicted error, $\mathbf{X}^{**} = \mathbf{X}^* + \mathbf{e}^*$. With NWP forecasts, **N**, being available, an optimal nonlinear hybrid forecast scheme, **S**, can be obtained which is comprised of the NWP product, **N**, and its error history, $\mathbf{e}_N$, the observed time series, **X**, and the error history, $\mathbf{e}_s$, gained by error recycling (using the nonlinear prediction scheme, $\mathbf{S}(\mathbf{N}, \mathbf{e}_N, \mathbf{X})$, iteratively). As joint forecasts for both the variable and its forecast error, $\mathbf{X}^*$ and $\mathbf{e}^*$, are possible, namely, $(\mathbf{X}^*, \mathbf{e}^*) = \mathbf{S}(\mathbf{N}, \mathbf{e}_N, \mathbf{X}, \mathbf{e}_s)$, predictions may also be amended by forecast error estimates, $\mathbf{X}^{**} = \mathbf{X}^* + \mathbf{e}^*$. This hybrid prediction may be regarded as basically a weather analogue model output statistics (A-MOS), which incorporates the local history of the observed and the predicted weather in an optimal nonlinear fashion utilizing error recycling. Further iterations, that is, predicting the error of the error forecat and repeating the process until convergence, may lead to more refined techniques of dynamical systems analysis. Applied to tropical cyclone tracks, the outcome of the monistic approach, $(\mathbf{X}^*, \mathbf{e}^*) = \mathbf{S}(\mathbf{N}, \mathbf{e}_N, \mathbf{X}, \mathbf{e}_s)$, demonstrated a significant improvement in the predictive skill of that model. The gain in skill achieved here is indeed large relative to the small amount of additional computational cost. It is hoped that this method of recycling forecast errors finds a similar practical application, both in tropical cyclone track and numerous other areas of weather forecasting.


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### References


Abarbanel HDI, Brown R, Sidorowick JH, Tsimring LS (1993) The analysis of observed chaotic data in physical systems. Reviews of Modern Physics 65: 1331–1392

Casdagli M (1989) Nonlinear prediction of chaotic time series. Physica D35: 335–356

Fraedrich K, Leslie LM (1987) Combining predictive schemes in short term forecasting. Mon Wea Rev 115: 1640–1644

Leslie LM, Fraedrich K (1990) Reduction of tropical cyclone position errors using an optimal combination of independent forecasts. Weather and Forecasting 5: 158–161

Leslie LM, Holland GJ, Glover F, Woodcock FJ (1990) A climatological-persistence (CLIPER) scheme for predicting Australian region tropical cyclone tracks. Aust Met Mag 38: 87–92

Leslie LM, LeMarshall JF (1998) Improved hurricane track forecasting from the continuous assimilation of high quality satellite wind data. Mon Wea Rev 126: 1248–1257

Neumann CJ (1972) An alternative to the HURRAN tropical cyclone forecast system. NOAA Tech Memo NWS SR-62

Mundell DB, Rupp JA (1995) Hybrid forecast aids at the Joint Typhoon Warning Centre: Applications and results. 21st Conf on Hurricanes and Tropical Meteorology (April 24–28), American Meteorological Society Miami Fla. 216–218

Sugihara G, May R (1990) Nonlinear forecasting as a way of distinguishing chaos from measurement error in a data series. Nature 344: 734–741

Thompson PD (1977) How to improve accuracy by combining independent forecasts. Mon Wea Rev 114: 228–229

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