NOTES AND CORRESPONDENCE

A Minimal Model for the Short-Term Prediction of Rainfall in the Tropics

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ABSTRACT

A "minimal" model is proposed here for the short-term prediction (up to 12 h ahead) of precipitation occurrence in the tropics. The model is purely statistical, consisting of an optimally weighted linear combination of a Markov chain and persistence. It is minimal in the sense that only surface data are needed, and the computing requirements are almost nil.

In this study the skill of the minimal model, i.e., accuracy relative to climatology and/or persistence, is demonstrated in theory and practice. The model was tested in real time during the 1986/87 Australian monsoon season at the tropical city of Darwin.

Results of the real-time experiment reveal that the minimal model was the only model of those available to the Australian Bureau of Meteorology (including manual forecasts, a regional NWP model, and a model output statistics (MOS) scheme) that exhibited forecast accuracy greater than that of both climatology and persistence.

1. Introduction

It has been pointed out elsewhere (for example Holland et al. 1987) that short-range forecasting in the tropics is often not as good as that obtained using climatology and/or persistence. The truth of this claim has been substantiated further by a recent real-time trial of methods for the short-term prediction of precipitation at the Australian tropical city of Darwin (Fraedrich and Leslie 1988). In this real-time trial, five methods were evaluated and compared with climatology and persistence. The methods included manual forecasts, the Australian region numerical weather prediction (NWP) model forecasts, model output statistics (MOS) schemes, a Markov chain model, and a linear combination of persistence and the Markov chain. Results of the real-time trial revealed that of all the schemes, only the Markov chain model and the Markov chain-persistence model showed a level of skill, i.e., accuracy, greater than both climatology and persistence. Moreover, the Markov chain model by itself was only marginally superior to climatology.

Two main findings emerged from the 1986/87 Darwin real-time trial. First, it showed that purely statistical models requiring only minimal resources were still clearly superior to other methods and, currently, were the only skillful methods. Second, the potency of the linear combination of the Markov chain model and persistence merited further investigation, in the sense that the meteorological and statistical background also needs to be analyzed.

The purpose of this note is to show that for tropical stations like Darwin, the most effective short-term forecasts of precipitation are obtainable from purely statistical methods like the Markov-persistence scheme. Deterministic methods may well prove to be the most accurate in the future but are currently inferior to the statistical schemes.

2. Combining probability forecasts

Two probabilistic forecast schemes, ϕ_1 and ϕ_2, predicting the binary variable, δ, (e.g., rainfall occurrence) can be linearly combined:

$$\phi_* = a\phi_1 + (1 - a)\phi_2, \qquad (2.1)$$

satisfying $0 \leqslant \phi_* \leqslant 1$. Minimizing the ensemble mean square error (or half-Brier score) of the combination forecast,

$$B_* = \langle(\delta - \phi_*)^2\rangle. \qquad (2.2)$$

One obtains the optimal weighting factor weight, a, of the combination forecast ϕ_* (2.1) and the related half-Brier score, B_*, after some algebra (Fraedrich and Leslie 1987):

$$a = \frac{\langle\delta\phi_1\rangle - \langle\delta\phi_2\rangle + \langle\phi_2^2\rangle - \langle\phi_1\phi_2\rangle}{\langle\phi_1^2\rangle + \langle\phi_2^2\rangle - 2\langle\phi_1\phi_2\rangle}$$

$$B_* = \langle(\delta - \phi_2)^2\rangle - \frac{(\langle\delta\phi_1\rangle - \langle\delta\phi_2\rangle + \langle\phi_2^2\rangle - \langle\phi_1\phi_2\rangle)^2}{\langle\phi_1^2\rangle + \langle\phi_2^2\rangle - 2\langle\phi_1\phi_2\rangle} \qquad (2.3)$$

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where $\langle(\delta - \phi_2)^2\rangle = B_2$ is the half-Brier score for the model ϕ_2; $\langle\phi_1\phi_2\rangle$ the covariance between the predictions ϕ_1 and ϕ_2, $\langle\delta\phi_1\rangle$ and $\langle\delta\phi_2\rangle$ are the covariances between either ϕ_1 and the observation δ or ϕ_2 and δ; $\langle\phi_1^2\rangle$ and $\langle\phi_2^2\rangle$ are measures of the prediction variances; and $\langle\delta^2\rangle$ is the observed variability. All of these second moments are not taken as deviations from their respective means. Two probabilistic forecast models of the same variable may then be combined optimally after evaluating the required statistics. The following two basic statistical schemes are combined to obtain an optimal and simply applicable model for the short-term prediction of rainfall during the tropical wet season: a Markov chain and persistence. Note that the combination of purely statistical schemes may be replaced by a simple model using multiple regression techniques for a probabilistic variable. However, possible scale interactions could remain unresolved.

a. Two rainfall forecast models

1) THE MARKOV CHAIN MODEL

The Markov chain model for short-term precipitation forecasts, originally developed by Fraedrich and Muller (1983) and applied by Miller and Leslie (1985) to a number of Australian cities, is based on immediately available 3 h single station surface observations. It was used to predict the probability of a binary (yes/no) observable (i.e., the occurrence of at least one rainfall event during the half-day period 0600 to 1800 LST). For the prediction of the probability of any precipitation, four mutually exclusive states (k) are used. These are three cloud-states (0–2, 3–5, and 6–8 oktas, all with no rain) and one rain state, which includes precipitation not only at a station, but also evidence of precipitation in the neighborhood (denoted by the international weather codes WW = 13–17, 20, 21, 25, 27, 50–69, 80–99). For the Markov chain model, the probability of precipitation for a given state k $(=1, \cdots, 4)$, at current time t, and for h $(=12\text{ h})$ ahead is

$$\phi_1 = \phi_m(k, t, h) = a(k, t, h) + \sum b_l(k, t, h)X_l \quad (2.4)$$

The four covariates X_l $(l = 1, \cdots, 4)$ are the surface pressure, the diurnally corrected 3 h pressure change, the dewpoint depression, and the east–west component of the wind, respectively. The introduction of other possible predictors from surface observations did not lead to a significant improvement in the forecast accuracy. The intercepts $a(k, t, h)$ are different for each month but common slopes are fitted for all months. The states k and covariates X_l are given by 3 h surface observations prior to the forecast period so that there is no lead time involved. These predictors can be interpreted as a single station mesoscale dataset with the Markov chain operating as a statistical mesoscale prediction model for the probability of rainfall.

2) PERSISTENCE FORECASTS

Persistence forecasts generally provide one kind of "zero skill" base from which one evaluates the quality of forecasts obtained by individual techniques. Here it is a binary (yes/no) forecast of the half-day (0600 to 1800 LST) observable, $\delta(t)$, using the previous half-day's observation, $(t - 24\text{ h})$, which is available with a 12 h lead time:

$$\phi_2 = \phi_p = \delta(t - 24\text{ h}). \quad (2.5)$$

Due to the time scale involved, persistence provides information on the synoptic scale, which is particularly useful in regions dominated by extended spells, such as monsoonal periods and breaks.

Thus, a linear combination of the Markov chain with persistence may be interpreted as a mesoscale model correcting the larger-scale forecasts.

b. Estimating the optimal weighting factor a

There are two methods for estimating the optimal value of a and the associated half-Brier score, B_*. First, the value of a may be derived from hindcasts using the long-term data records at the station. Direct substitution into (2.3) will then provide the optimal values for a and B_*. Alternatively, a first-order estimate of the value of a may be obtained theoretically, as described in the Appendix. If it is assumed that the second moments are dominated by the means (and not the deviations) then, for the Markov chain–persistence combination, the optimal values of a and B_* are found to be [Eq. (A6) in the Appendix]

$$a \sim \frac{1}{2} + (B_p - B_m)/2B_c,$$

$$B_* \sim B_p - a^2 B_c,$$

where B_p, B_m and B_c are the Brier scores of persistence, the Markov chain, and climatology, respectively.

The results of a precipitation forecast trial (Fraedrich and Leslie 1987) for the Australian tropical station Darwin provide a check of the value of the optimal weighting factor, a, as will be shown in section 3.

Persistence–climatology, as an optimized linear combination of forecast schemes, defines the level of zero-skill for combined predictions. The optimal values for the weighting factor a and, in particular, the half-Brier score B_* (Fraedrich and Leslie 1988, Eq. 14 with $\phi_1 = \phi_p$),

$$a = 1 - B_p/2B_c$$

$$B_* = B_1 - B_1^2/4B_c$$

are also discussed in section 3.

3. Results from the 1986/87 real-time trial

Half-Brier scores were computed for a number of schemes in the real-time trial for the monsoon season

TABLE 1. Half-Brier score for the various schemes are shown for the real-time trial of short-term rainfall prediction at Darwin. Also included are the half-Brier scores for climatology and persistence. The half-Brier scores are given for both WW-rain and RR-rain (in parentheses). Smaller half-Brier scores indicate greater accuracy.

Method	Half-Brier score
Manual forecasts	0.187 (0.226)
NWP model	0.461 (0.539)
MOS	0.221 (0.223)
Markov chain	0.147 (0.173)
Markov chain-persistence	0.127 (0.167)
Climatology	0.153 (0.216)
Persistence	0.222 (0.356)
Persistence–climatology	0.142 (0.209)

December 1986 to February 1987. The results are described in detail by Fraedrich and Leslie (1988) and only a few relevant results are needed here.

In Table 1, the performance of five forecasting techniques is summarized. These include the manual forecasts, numerical weather prediction model forecasts, model output statistics, Markov chain forecasts, and the combined persistence–Markov chain forecasts. They are compared with climatology, persistence and persistence–climatology, which may be regarded as levels of zero skill. The results are presented for both WW- and RR-rainfall (using the International Weather Code notation). It is immediately clear from Table 1 that the only prediction scheme which is significantly more accurate than climatology, persistence, and persistence–climatology is the Markov–persistence combination.

The optimal weighting factor, a, for the Markov–persistence combination was chosen from the 20-yr 1962–1981 surface dataset. The value of a, computed from (2.3), was $a = 0.66$ for WW-rain and 0.90 for RR-rain. The Appendix also provides for an estimate of the value of a, from (A6), of $a = 0.74$ (0.92), using the 1986/87 data. This close correspondence between the estimates of a, using the general theory of Fraedrich and Leslie (1987) and the first-order estimate in the Appendix, gives added confidence in the value of a and its stability from season to season.

Why is the combination of persistence and the Markov chain so effective? Part of the answer lies in Fig. 1, which shows the observed half-day (0600 to 1800 LST) rainfall for Darwin during the period 1 December 1986–28 February 1987. Also shown by broken heavy lines are the periods during which measurable precipitation at the station was recorded and the periods during which measurable rainfall or evidence of rainfall near the station was recorded. Particularly in the case of WW-rain, there are extended wet and dry spells, characteristic of a monsoon regime. In simplistic terms, it appears that the role of persistence is to represent the synoptic scale and the Markov chain, which uses observed data at the time the forecast is made to provide the embedded mesoscale information.

4. Conclusions

It has been demonstrated, both in theory and in practice, that a minimal model consisting of a optimally

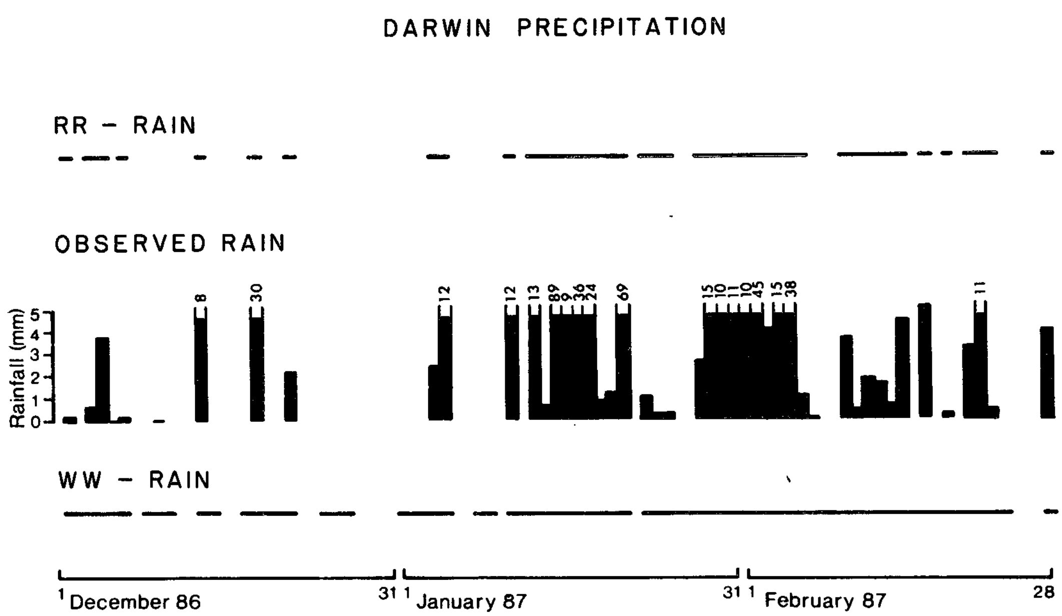

FIG. 1. The observed half-day (0600 to 1800 LST) rainfall at Darwin for the period 1 December 1986 to 28 February 1987 is shown (in mm). Also represented schematically by heavy lines are the observed half-days on which RR- and WW-rainfall was reported.

weighted linear combination of persistence, and a Markov chain fitted to 3 h surface data, is the most skillful of all the models tested in a real-time trial of short-term rainfall prediction for the Australian tropical station of Darwin. In fact, the Markov–persistence model was the only model that exhibited accuracy significantly greater than that of climatology.

The results of the trial and the theory contained in the Appendix suggest that such a "minimal" model should prove to be an effective forecasting tool for tropical stations like Darwin, which have reasonably extended spells of dry and wet weather. The attractiveness of the model is enhanced by the fact that only surface data is needed and the computing resources required are very small.

It is reasonable to anticipate that deterministic methods, such as dynamical weather prediction models developed specifically for the tropics, will eventually reach levels of skill beyond these of the statistical methods. In the meantime, however, such models as the one described here appear to be essential for a skillful short-term forecast of precipitation in the tropics.

APPENDIX

Estimation of Weighting Factor *a*

The required statistical parameters (2.3) for the combination forecast (2.1) are estimated as follows. They can be derived in terms of the error of each individual scheme. Thus, from the half-Brier scores of each forecast scheme, ϕ_i with $i = m$ or p, one can deduce the covariance between its prediction and the observation:

$$B_i = \langle(\delta - \phi_i)^2\rangle = \langle\delta^2\rangle + \langle\phi_i^2\rangle - 2\langle\delta\phi_i\rangle$$

$$\langle\delta\phi_i\rangle = (\phi_c + \langle\phi_i^2\rangle - B_i)/2. \qquad \text{(A1)}$$

First, the statistics of predictions based on the climate probability are determined for further reference and comparison. The climate probability, ϕ_c, to forecast the binary observable, $\delta = 1$ or 0 is given by the mean, $\langle\delta\rangle = \langle\delta^2\rangle = \phi_c$; the half-Brier score and the second moments of the climate forecasts are then

$$B_c = \phi_c - \phi_c^2$$

$$\langle\delta\phi_c\rangle = \phi_c\langle\delta\rangle = \langle\phi_c^2\rangle = \phi_c^2. \qquad \text{(A2)}$$

Next consider persistence, $\phi_i = \phi_p$, as a binary prediction model for the binary variable δ. The required statistical relationships are readily deduced:

$$B_p = 2(\phi_c - \langle\delta\phi_p\rangle)$$

$$\langle\delta\phi_p\rangle = \langle\delta(t)\delta(t - 24\text{ h})\rangle = \phi_c - B_p/2$$

$$\langle\phi_p\rangle = \langle\phi_p^2\rangle \doteq \phi_c. \qquad \text{(A3)}$$

Note that the mean of the original time series, $\langle\phi\rangle = \phi_c$, and the mean of a persistence forecast, say from one day to the next, $\langle\phi_p\rangle$, are identical because the two time series differ only by a time shift.

For categorical predictions (such as persistence of the rainfall occurrence), the half-Brier score B_p is identical to the relative number of incorrect forecasts.

Finally, the Markov chain, $\phi_i = \phi_m$, is introduced as a probabilistic forecast scheme calibrated by past weather observations. The half-Brier score and the second moment are

$$B_m = \langle\delta^2\rangle + \langle\phi_m^2\rangle - 2\langle\delta\phi_m\rangle$$

$$\langle\delta\phi_m\rangle = (\phi_c + \langle\phi_m^2\rangle - B_m)/2. \qquad \text{(A4a)}$$

One may assume that there will be good reliability (or small bias) for a forecast scheme like the Markov chain, if it is based on statistically stable estimates from a sufficiently large dataset (i.e., $\langle\phi_m\rangle \sim \langle\phi_c\rangle$). Furthermore, one may assume that if the second moment is dominated by the mean and not the deviations, then

$$\langle\phi_m^2\rangle = \langle\phi_m\rangle^2 + \langle\phi_m'^2\rangle \sim \phi_c^2. \qquad \text{(A4b)}$$

This assumption should only be made if the mean, $\langle\phi_m\rangle \sim \phi_c$ is large, i.e., the rainfall occurrence is the dominating mode of the time series for which persistence provides relatively good forecasts. The last statistical parameter to be determined for the combination forecast is the covariance between the predictive schemes, ϕ_1 and ϕ_2:

$$\langle\phi_m\phi_p\rangle = \langle\phi_m\rangle\langle\phi_p\rangle + \langle\phi_m'\phi_p'\rangle \sim \langle\phi_m\rangle\langle\phi_p\rangle. \qquad \text{(A5)}$$

Again, as a first estimate, one may assume the second moment to be dominated by the means and neglect the contribution by the deviations, i.e., the second term in (A5).

Introducing (A2) to (A5) into (2.3) leads to first-order estimates of the weight of the Markov-persistence combination and its half-Brier score:

$$a \sim \frac{1}{2} + (B_p - B_m)/2B_c$$

$$B_* \sim B_p - a^2B_c. \qquad \text{(A6)}$$

A special case arises when a climate-persistence combination is considered. Replacing ϕ_m by ϕ_c, B_m by B_c, etc., yields the following weight and error (note that they are not approximated):

$$a = B_p/2B_c$$

$$B_* = B_c(1 - (1 - a)^2). \qquad \text{(A7)}$$

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