contemporary Charles Babbage, whose interpretation of intelligence (if not life itself) was more formalized, abstract, and mechanical. The mechanization of the mind continued with George Boole's logical algebra and then the work of Alan Turing and John von Neumann on automating thought processes, which led directly to the invention of the digital computer and the beginnings of artificial intelligence. It was a similar inquiry into the abstract nature of life, as distinct from mind, that prompted von Neumann's investigations into self-replicating machinery and Turing's work on embryogenesis and "unorganized machines" (related to neural networks).

Formal Methods

While complexity theory is concerned with the manner in which complex behavior arises from simple systems, AL is interested in how systems generate continually increasing levels of complexity. The most striking feature of living systems is their ability to self-organize and self-maintain—a property that Humberto Maturana and Francisco Varela have termed "autopoiesis" (Maturana & Varela, 1980). Evolution, embryogenesis, learning, and the development of social organizations are therefore the mechanisms of primary interest to AL researchers.

The key features of AL models are the use of populations of semi-autonomous entities, the coupling of these through simple local interactions (no centralized control and little or no globally accessible information), and the consequent emergence of collective, persistent phenomena that require a higher level of description than that used to describe their substrate. Conventional mathematical notation is not usually appropriate for such distributed and labile systems, and the individual computer programs are often their own best description. There are, however, a number of frequently used abstract structures and formal grammars, including the following:

Cellular automata, in which the populations are arrays of finite state machines and interactions occur between neighboring cells according to simple rules. Under the right conditions, emergent entities (such as the glider in John Conway's Game of Life) arise and persist on the surface of the matrix, interacting with other entities in computationally interesting ways.

Genetic algorithms, in which the populations are genomes in a gene pool and interactions occur between their phenotypes and some form of stressful environment. Natural selection (or sometimes human choice) drives the population to adapt and grow ever fitter, perhaps solving real practical problems in the process.

L-systems, or *Lindenmayer systems*, which provide a grammar for defining the growth of branching (often plant-like) physical structures, as insights into morphology and embryology.

Autonomous agents, which are composite code and data objects, representing mobile physical entities (robots, ants, stock market traders) embedded in a real or simulated environment. They interact locally by sensing their environment and receiving messages from other agents, giving rise to emergent phenomena of many kinds including cooperative social structures, nest-building, and collective problem-solving.

Autocatalytic networks, in which the populations are of simulated enzymes and the interactions are equivalent to catalysis. Such networks are capable of self-generation and a growth in complexity, mimicking the bootstrapping process that presumably gave rise to life on Earth.

Current Status

Like most new fields, AL has undergone cycles of hubris and doubt, innovation and stasis, and differentiation and consolidation. The listing of topics for the latest in the series of workshops started by Langton in 1987 is as broad as ever, although probably the bulk of AL work today (2004) is focused on artificial evolution. Most research concentrates on fine details, while the basic philosophical questions remain largely unanswered. Nevertheless, AL remains one of relatively few fields where one can ask direct questions about one's own existence in a practical way.

STEVE GRAND

See also **Catalytic hypercycle; Cellular automata; Emergence; Game of life; Hierarchies of nonlinear systems; Turing patterns**

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ASSEMBLY OF NEURONS

See **Cell assemblies**

ATMOSPHERIC AND OCEAN SCIENCES

Earliest works on the study of the atmosphere and ocean date back to Aristotle and his student Theophrastus in 350 BC and further progressed through Torricelli's invention of the barometer in 1643, Boyle's law in 1657, and Celsius's invention of the thermometer in 1742 (due to Galileo in 1607). The first rigorous theoretical model for the study of the atmosphere was proposed by

Vilhelm Bjerknes in 1904, following which many scientists began to apply fundamental physics to the atmosphere and ocean. The advent of these theoretical approaches and the invention of efficient communication technologies in the mid-20th century made numerical weather prediction feasible and was in particular encouraged by Lewis Fry Richardson and John von Neumann in 1946, using the differential equations proposed by Bjerknes. Today, advanced numerical modeling and observational techniques exist, which are constantly being developed further in order to understand and study the complex nonlinear dynamics of the atmosphere and ocean.

This overview article summarizes the governing equations used in atmospheric and ocean sciences, features of atmosphere–ocean interaction, and processes for an idealized geometry and structure with reference to a one-dimensional vertical scale (Figure 1), a two-dimensional vertically averaged scale (Figures 3(a) and 4(a)), a two-dimensional zonally averaged meridional scale (Figures 3(b) and 4(b)), and a three-dimensional scale (Figure 2), and regimes of interacting systems (such as El Niño and Southern Oscillation and North Atlantic Oscillation) (Figures 5–7). The entry serves as an introduction to the many nonlinear processes taking place (for example, chaos, turbulence) and provides a few illustrative examples of self-organizing coherent structures of the nonlinear dynamics of the atmosphere and ocean.

Governing Equations

The combined atmosphere and ocean system can be regarded as a huge volume of fluid resting on a rotating oblate spheroid with varying surface topography moving through space, with an interface (which in general is discontinuous) between two fluid masses of differing densities. This coupled atmosphere-ocean system is driven by energy input through solar radiation (see Figure 1), gravity (for example, through interaction with other stellar bodies such as the Sun and Moon, i.e., tides), and inertia. The entire fluid is described by equations for conserved quantities such as momentum, mass (of air, water vapor, water, salt), and energy together with equations of state for air and water (*See* **Fluid dynamics; Navier–Stokes equation**). The movement of large water or air masses in a rotating reference frame adds to the complexity of motions, due to the presence of Coriolis forces, introduced by Coriolis in 1835.

Atmosphere-ocean interactions can be defined as an exchange of momentum, heat, and water (vapor and its partial masses: salts, carbon, oxygen, nitrogen, etc.) between air and water masses. The governing equations in the Euler formulation and a cartesian coordinate system are given by:

(i) The conservation of momentum

$$\frac{\mathrm{d}\boldsymbol{u}}{\mathrm{d}t} + 2\Omega \times \boldsymbol{u} = -\frac{1}{\rho}\nabla p - \boldsymbol{g} + \boldsymbol{F}_{\text{ext}} + \boldsymbol{F}_{\text{fric}}, \quad (1)$$

where the second term $2\Omega \times \boldsymbol{u}$ is the term due to the Coriolis force (Ω is the angular velocity of the Earth; $|\Omega| = 7.29 \times 10^{-5}\ \mathrm{s}^{-1}$), and forces due to a pressure gradient ∇p, gravity ($|\boldsymbol{g}| = 9.81\,\mathrm{m\,s}^{-2}$) and external ($\boldsymbol{F}_{\text{ext}}$) as well as frictional ($\boldsymbol{F}_{\text{fric}}$) forces are included. Note that the operator $\mathrm{d}/\mathrm{d}t$ is defined by

$$\frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}t} = \left(\frac{\partial}{\partial t} + \boldsymbol{v}\cdot\nabla\right)\boldsymbol{v}.$$

(ii) The conservation of mass (or continuity equation)

$$\frac{1}{\rho}\frac{\mathrm{d}\rho}{\mathrm{d}t} + \nabla\cdot\boldsymbol{u} = 0. \quad (2)$$

Note that there are alternative formulations such as the Lagrangian and impulse-flux form for these equations, and cartesian coordinate systems can be mapped to different geometries such as spherical coordinates by appropriate transformations.

(iii) The conservation of energy (First Law of Thermodynamics) and Gibbs's equation (Second Law of Thermodynamics)

$$\begin{aligned}\frac{\mathrm{d}Q}{\mathrm{d}t} &= \frac{\mathrm{d}\varepsilon}{\mathrm{d}t} + p\frac{\mathrm{d}\alpha}{\mathrm{d}t},\\ \frac{\mathrm{d}\eta}{\mathrm{d}t} &= \frac{1}{T}\frac{\mathrm{d}\varepsilon}{\mathrm{d}t} + \frac{p}{T}\frac{\mathrm{d}\alpha}{\mathrm{d}t} - \sum\frac{\mu_i}{T}\frac{\mathrm{d}\gamma_i}{\mathrm{d}t},\end{aligned} \quad (3)$$

where Q is the heat supply (sensible, latent, and radiative heat fluxes; see Figure 1), T is the temperature, ε the internal energy and α the specific volume ($\alpha = 1/\rho$), η the entropy, μ_i the chemical potentials, and γ_i the partial masses. The conservation of energy states in brief that the change in heat is balanced by a change in internal energy and mechanical work performed, and Gibbs's equation determines the direction of an irreversible process, relating entropy to a change in internal energy, volume, and partial masses.

(iv) The conservation of partial masses of water and air, that is, salinity for water, where all constituents are represented as salts and water vapor for air, yield equations similar to (2)

$$\frac{1}{\rho_{\mathrm{v}}}\frac{\mathrm{d}\rho_{\mathrm{v}}}{\mathrm{d}t} + \nabla\cdot\boldsymbol{u} = W_v$$

and

$$\frac{\mathrm{d}\rho s}{\mathrm{d}t} + \rho s\nabla\cdot\boldsymbol{u} = W_{\mathrm{s}}, \quad (4)$$

where ρ_{v} is the density of water vapor, s the specific salinity (gram salts per gram water), and W_v, W_s

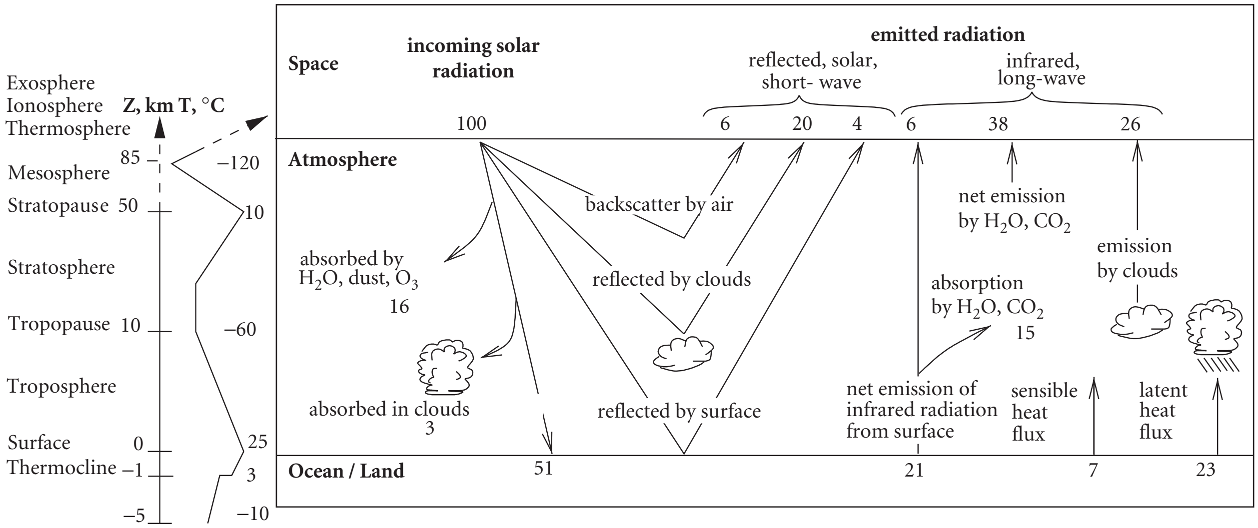

Figure 1. Sketch of the vertical structure of the atmosphere–ocean system and radiation balance and processes in the global climate system. Adapted from National Academy of Sciences (1975). Note that lengths are not to scale and and temperatures indicate only global averages.

contain possible source and sink terms as well as the effect of molecular diffusion in terms of the concentration flux density $\boldsymbol{S}\,(-\nabla\cdot\boldsymbol{S})$ and possible phase changes.

(v) The equation of state for a mixture of salts and gases for air and water, whose constituent concentrations are virtually constant in the atmosphere and ocean is

$$p \approx \rho R T\,(1 + 0.6078\,q)\,, \qquad (5)$$

where R is the gas constant for dry air ($R = 287.04\,\mathrm{J\,kg^{-1}\,K^{-1}}$) and $q = \rho_v/\rho$ is the specific humidity. Similarly, the equation of state for near incompressible water is

$$\rho \approx \rho_0[1 - \alpha(T - T_0) + \beta(S - S_0)], \qquad (6)$$

where ρ_0, T_0, and S_0 are reference values for density, temperature, and salinity ($\rho_0 = 1028\,\mathrm{kg\,m^{-3}}$, $T_0 = 283\,\mathrm{K}(= 10°\mathrm{C})$, $S_0 = 35‰$), and α and β are the coefficients of thermal expansion and saline contraction ($\alpha = 1.7 \times 10^{-4}\,\mathrm{K^{-1}}$, $\beta = 7.6 \times 10^{-4}$), see Krauss (1973); Cushman-Roisin (1993).

Equations detailed in (i)–(v) form a set of hydrothermodynamic equations for the atmosphere-ocean system to which various approximations and scaling limits can be applied. Among them are the shallow-water equations, primitive equations, the Boussinesq and anelastic approximation, quasigeostrophic, and semigeostrophic equations and variants or mixtures of these. These equations have to be solved with appropriate boundary conditions and conditions at the air-sea interface; for details refer to Krauss (1973), Gill (1982) and Kraus & Businger (1994). For studies of the upper atmosphere, further equations for the geomagnetic field can also be taken into account (Maxwell's equations).

Atmospheric Structure and Circulation

In the vertical dimension, several atmospheric layers can be differentiated (see Figure 1). Figure 2 gives the length and time scales of typical atmospheric processes.

From sea level up to about 2 km is the atmospheric boundary layer, characterized by momentum, heat, moisture, and water transfer between the atmosphere and its underlying surface. Above the boundary layer is the troposphere (Greek, *tropos* meaning *turn, change*) that constitutes most of the total mass of the atmosphere (about 10 km height) and is largely in hydrostatic balance characterized by a decrease in temperature. Above the troposphere and stratosphere, which contains the ozone layer, temperatures rise throughout. The mesosphere, which is bounded by the stratopause (about 50 km height) below and mesopause (about 85 km height) above, is a layer of very thin air where temperatures drop to extreme lows. Above the mesopause, temperatures increase again throughout the thermosphere (from about 85 km to 700 km), the largest layer of the atmosphere, where the ionosphere is located (between about 100 km and 300 km). The ionosphere contains ionized atoms and free electrons and permits the reflection of electromagnetic waves. Above the thermosphere is the exosphere, which is the outermost layer of the atmosphere and the transition region between the atmosphere and outer space, the magnetosphere in particular, where atoms can escape into space beyond the so-called escape velocity and where the Van Allen belt is situated.

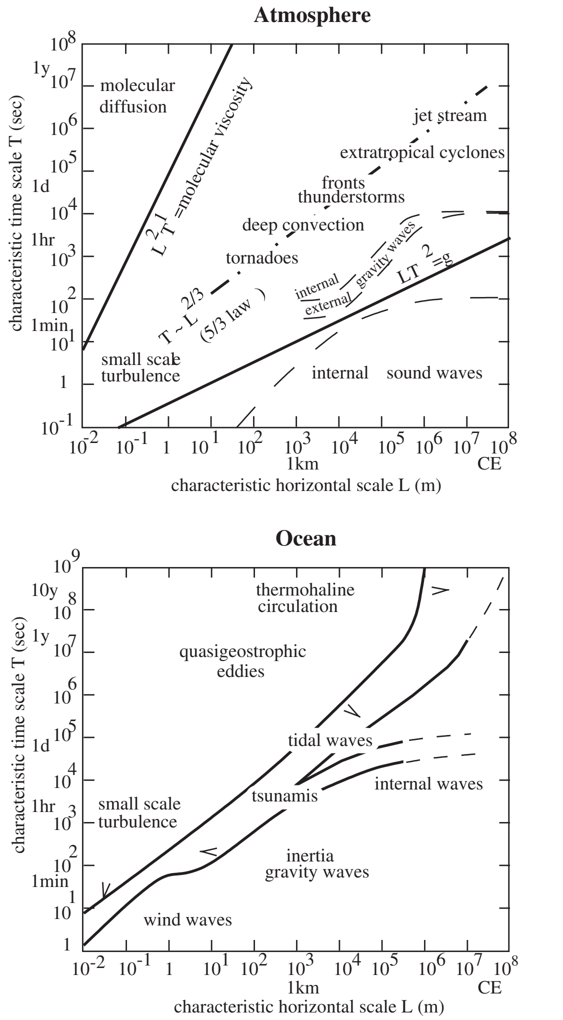

Figure 2. Schematic logarithmic time and horizontal length scales of typical atmospheric and oceanic phenomena. Note that Richardson's $L \propto T^{3/2}$ relation and CE stands for circumference of the Earth. Modified from Lettau (1952), Smagorinsky (1974), and World Meteorological Organization (1975).

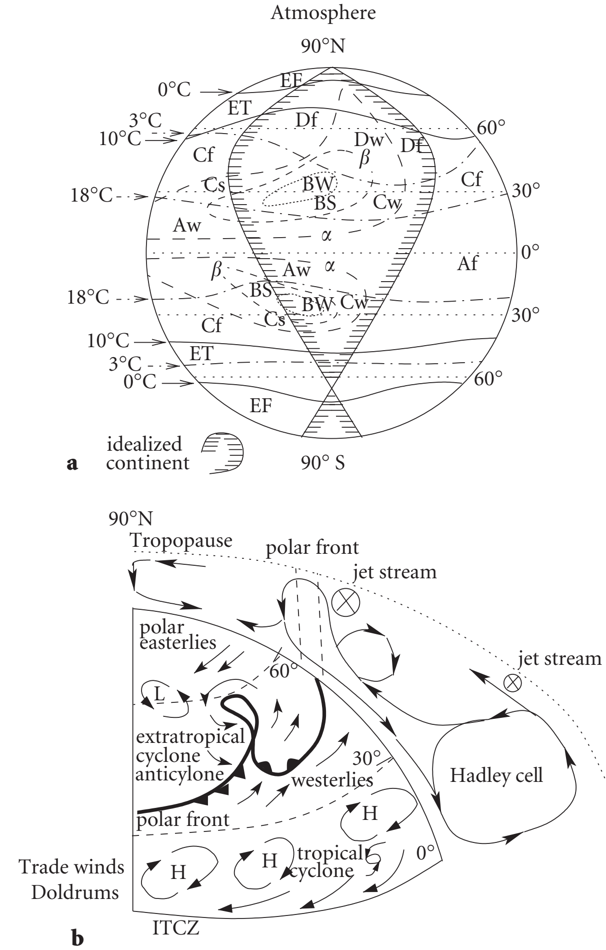

Figure 3. Sketch of the near-surface climate and atmospheric circulation of the Earth with an idealized continent. (a) Averaged isothermals of the coldest month (dashed-dot, −3°C, 18°C) and the warmest month (solid, 0°C, 10°C) and periodical dry season boundaries α and dry climate β. The following climate regions are indicated: wet equatorial climate (Af), tropical wet/dry climate (Aw), desert climate (BW), steppe climate (BS), sinic climate (Cw), Mediterranean climate (Cs), humid subtropical climate (Cf), humid continental climate (Df), continental subarctic climate (Dw), tundra climate (ET) and snow and ice climate (EF). Modified from Köppen (1923). (b) The zonal mean jet streams (primary circulation) and mass overturning (secondary circulation) in a meridional height section, the subtropical highs (H) and subpolar lows (L), polar easterlies, westerlies, polar front, trade winds, and intertropical convergence zone (ITCZ). ▲ denotes a cold front and ● a warm front. Adapted from Palmen (1951), Defant and Defant (1958), and Hantel in Bergmann & Schäfer (2001).

A low (high) in meteorology refers to a system of low (high) pressure, a closed area of minimum (maximum) atmospheric pressure (closed isobars, or contours of constant pressure) on a constant height chart. A low (high) is always associated with (anti)cyclonic circulation, thus also called a cyclone (anticyclone). Anticyclonic means clockwise in the Northern Hemisphere (and counterclockwise in the Southern Hemisphere). Cyclonic means counterclockwise in the Northern Hemisphere (and clockwise in the Southern Hemisphere). At zeroth order, a balance of pressure gradient forces and Coriolis forces, that is, geostrophic balance, occurs, leading to the flow of air along isobars instead of across (in the direction of the pressure gradient). A front is a discontinuous interface or a region of strong gradients between two air masses of differing densities or temperatures, thus encouraging conversion of potential into kinetic energy (examples are polar front, arctic front, cold front, and warm front).

Hurricanes and typhoons (local names for tropical cyclones) transport large amounts of heat from low to mid and high latitudes and develop over oceans. Little is known about the initial stages of their formation, although they are triggered by small low-pressure systems in the Intertropical Convergence Zone (*See* **Hurricanes and tornadoes**). Because of their strong winds, cyclones are particularly active in inducing

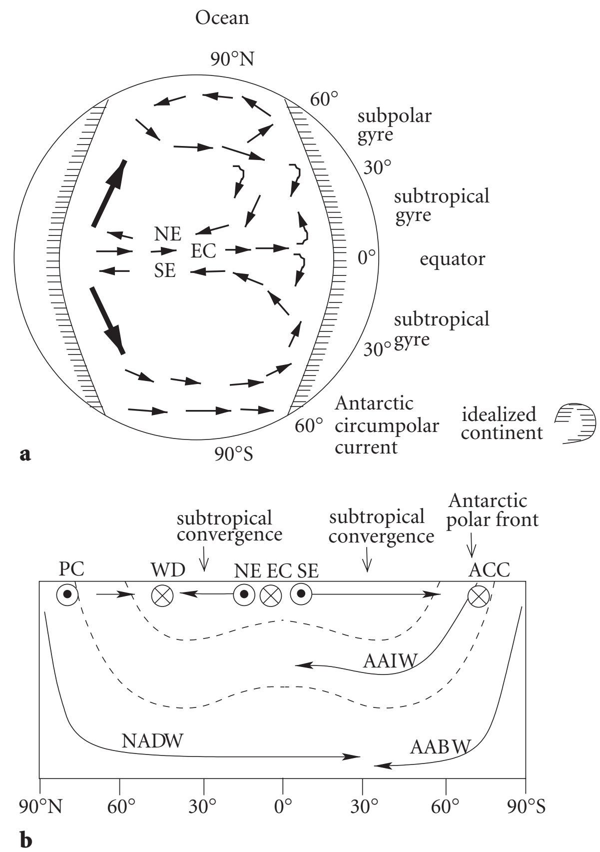

Figure 4. Sketch of the oceanic circulation of an idealized basin (a) the global wind-induced distribution of ocean currents (primary circulation) and (b) the zonal mean thermohaline circulation (secondary circulation) in a meridional depth section showing upper, intermediate, deep and bottom water masses. NE denotes the north equatorial, EC the equatorial and SE the south equatorial current. PC stands for polar current, ACC for Antarctic circumpolar current, WD for west drift, AAIW for Antarctic intermediate waters, NADW for North Atlantic deep water, and AABW for Antarctic bottom water. Adapted from Hasse and Dobson (1986).

upwelling and wind-driven surface water transport. Extratropical cyclones are frontal cyclones of mid to high latitudes (see Figures 2 and 3).

Meteorology and oceanography are concerned with understanding, predicting, and modeling the weather, climate, and oceans due to their fundamental socioeconomic and environmental impact. In meteorology, one distinguishes between short (1–3 days) and medium-range (4–10 days) numerical weather prediction models (NWPs) for the atmosphere and general circulation models (GCMs, *See* **General circulation models of the atmosphere**). While NWPs for local and regional weather prediction are usually not coupled to ocean models, GCMs are global three-dimensional complex coupled atmosphere-ocean models (which even include the influence of land masses), used to study global climate change, modeling radiation, photochemistry, transfer of heat, water vapor, momentum, greenhouse gases, clouds, ocean temperatures, and ice boundaries. The atmosphere-ocean interface couples the “fast” processes of the atmosphere with the comparably “slow” processes of the ocean through evaporation, precipitation, and momentum interaction. GCMs are validated using statistical techniques and correlated to the actual climate evolution. Additionally, the application of GCMs to different planetary atmospheres, for example, on Mars and Jupiter, leads to a greater understanding of the planet’s history and environment.

The complexity of the dynamics of the atmosphere and ocean is largely due to the intrinsic coupling between these two large masses at the air-sea interface.

Ocean

Ocean circulation is forced by tidal forces (also known to force atmospheric tides), due to gravitational attraction, wind stress, applied shear forces acting on the interface, and external, mainly solar, radiation, penetrating into the sea surface and affecting the heat budget and water mass due to evaporation. Primary sources of tidal forcing, earliest work on which was undertaken by Pierre-Simon Laplace in 1778, are the Moon and the Sun. One discerns between diurnal, semidiurnal, and mixed-type tides.

In the ocean, one distinguishes between two types of ocean currents: surface (wind-driven) and deep circulation (thermohaline circulation). Separating the surface and deep circulation is the thermocline, a small layer of strong gradient of temperature, salinity, and density, acting as an interface between the two types of circulations.

Surface circulation ranging up to 400 m in depth is forced by the prominent westerly winds in the mid-latitudes and trade winds in the tropical regions (see Figures 3 and 4), which are both forced by solar heating and Coriolis forces leading to expansion of water near the equator and decreased density, but increased salinity due to evaporation. An example of the latter is the Gulf Stream in the North Atlantic. The surface wind stress, solar heating, Coriolis forces, and gravity lead to the creation of large gyres in all ocean basins with clockwise (anticyclonic) circulation in the northern hemisphere and counterclockwise circulation in the southern hemisphere. The North Atlantic Gyre, for example, consists of four currents: the north equatorial current, the Gulf Stream, the North Atlantic current, and the Canary current.

Ekman transport, the combination of wind stress and Coriolis forces, leads to a convergence of water masses in the center of such gyres, which increases the sea surface elevation. The layer of Ekman transport can be 100–150 m in depth and also leads to upwelling due to conservation of mass on the western (eastern) coasts for winds from the north (south) in the Northern (Southern) Hemisphere. As a consequence, nutrient-

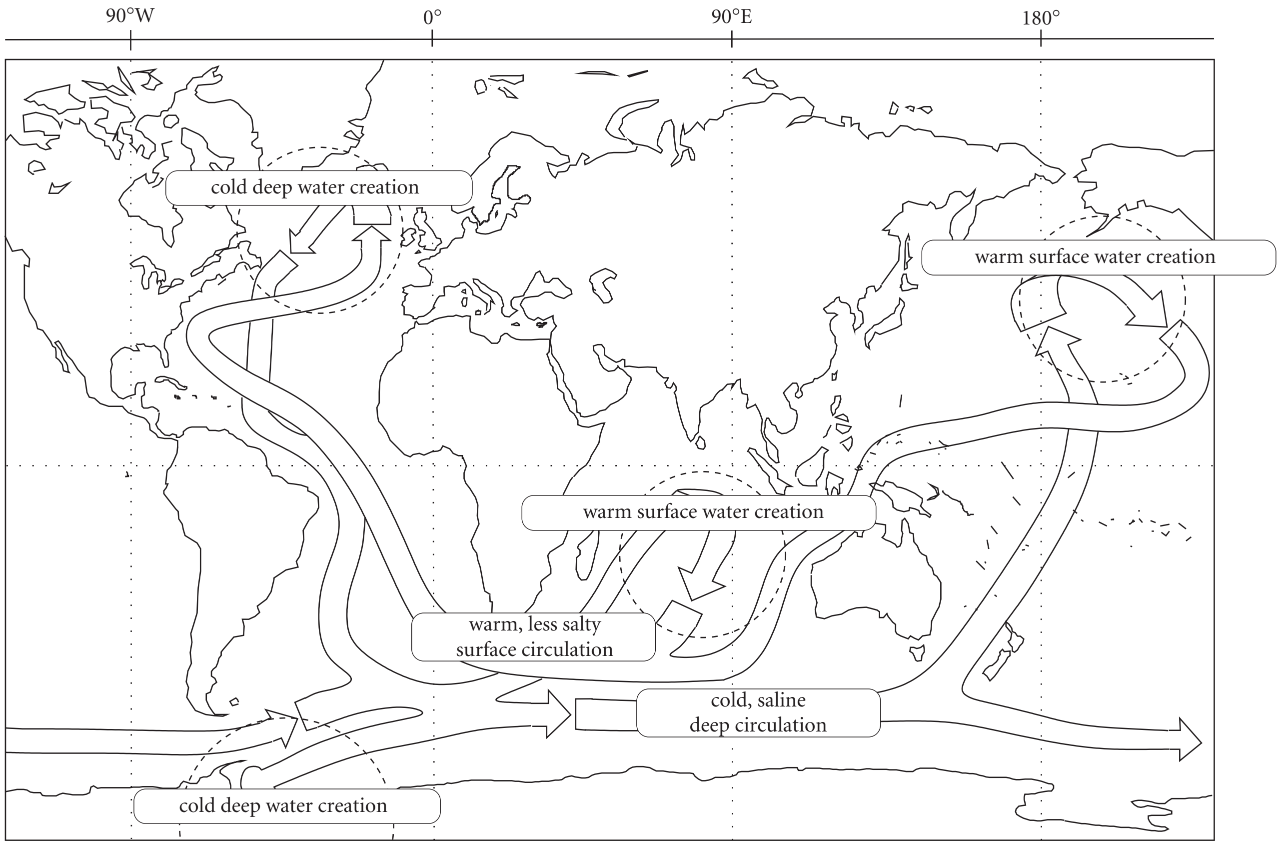

Figure 5. Sketch of the global conveyor belt through all oceans, showing the cold saline deep circulation, the warm, less salty surface circulation, and the primary regions of their creation. Note that this circulation is only characteristic of the actual global circulation. Adapted from Broecker (1987).

rich deep water is brought to the surface. With the opposite wind direction, Ekman transport acts to induce downwelling.

Another important combination of forces is the balance of Coriolis forces and gravity (pressure gradient forces), which is called geostrophic balance, leading to the movement of mass along isobars instead of across (geostrophic current), similar to the atmosphere. The boundary currents along the eastern and western coastlines are the major geostrophic currents in a gyre. The western side of the gyre is stronger than the eastern due to the Earth's rotation, called western intensification.

Deep circulation makes up 90% of the total water mass and is driven by density forces and gravity, which in turn is a function of temperature and salinity. High-density deep water originates in the case of extreme cooling of the sea surface in the polar regions, sinking to large depths as a density current, a strongly nonlinear phenomenon. When the warm Gulf Stream waters, which have increased salinity due to excessive evaporation in the tropics, move north due to the North Atlantic Gyre, they are cooled by Arctic winds from the north and sink to great depths forming the high-density Atlantic deep waters (see Figure 5). The downward transport of water is balanced by upward transport in low- and mid-latitude regions.

The most prominent example of the interaction between atmospheric and ocean dynamics is the global conveyor belt, which links the surface (wind-driven) and deep (thermohaline) circulation to the atmospheric circulation. The global conveyor belt is a global circulatory system of distinguishable and recognizable water masses traversing all oceans (see Figure 5). The water masses of this global conveyor belt transport heat and moisture, contributing to the climate globally. In Earth's history, the global conveyor belt has experienced flow reversals and perturbations leading to changes in the global circulatory system. The rather recent anthropogenic impact on climate and oceans through greenhouse gas emissions has the potential to create instability in this large-scale dynamical system, which could alter Earth's climate and have devastating environmental and agricultural effects.

ENSO and NAO

Another example of atmosphere-ocean coupling is the combination of the El Niño and Southern Oscillation (ENSO). The El Niño ocean current (and associated

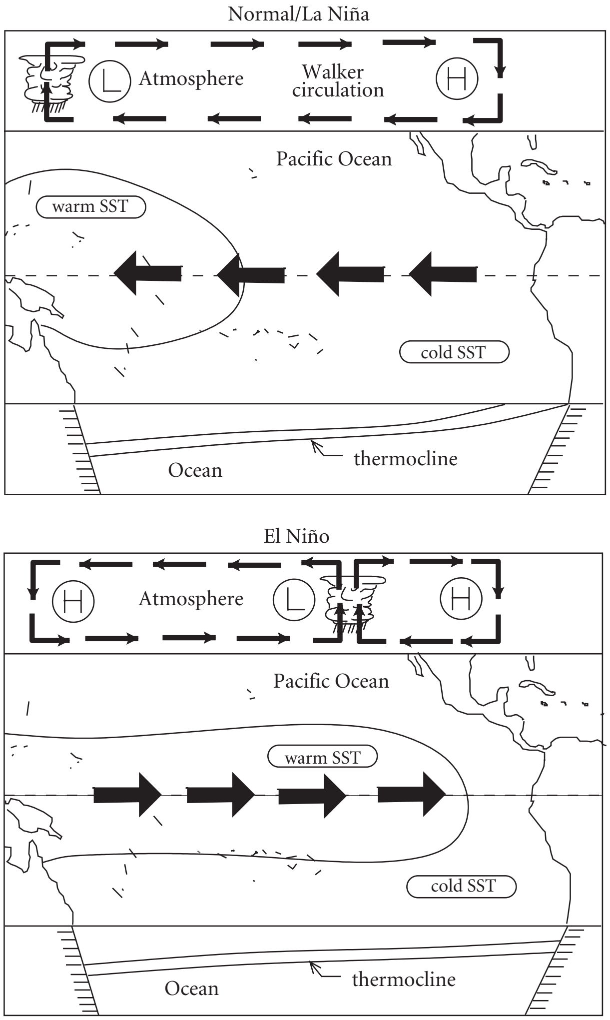

Figure 6. Sketch of the El Niño in the tropical Pacific, showing a reversal in (trade) wind direction from easterlies to westerlies during an El Niño period bringing warmer water (warm corresponds to a positive sea-surface temperature [SST]) close to the South American coast, displacing the equatorial thermocline downwards. Note the change in atmospheric tropical convection and associated heavy rainfall. After McPhaden, NOAA/TAO (2002) and Holton (1992).

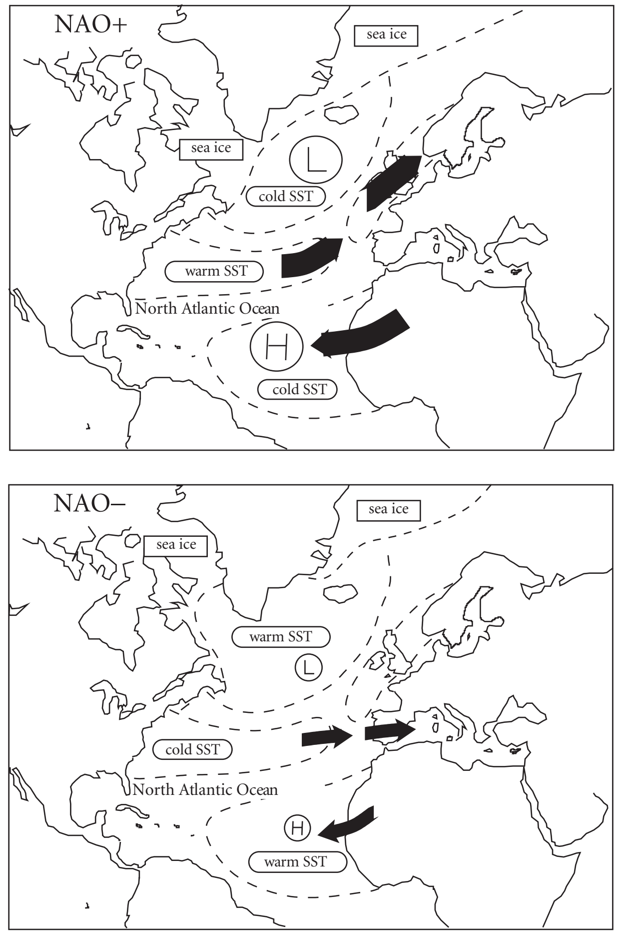

Figure 7. Sketch of the North Atlantic Oscillation (NAO) during the northern hemisphere winter season. Positive NAO (NAO+) showing an above-usual strong subtropical high-pressure center and subpolar low, resulting in increased wind strengths and storms crossing the Atlantic towards northern Europe. NAO+ is associated with a warm wet winter in Europe and cold dry winter in North America. Central America experiences mild wet winter conditions. Negative NAO (NAO−) shows a weaker subtropical high and subpolar low, resulting in lower wind speeds and weaker storms crossing the Atlantic toward southern Europe and receded sea ice masses around Greenland. NAO− is associated with cold weather in northern Europe and moist air in the Mediterranean. Central America experiences colder climates and more snow. Adapted from Wanner (2000).

wind and rain change) is named from the Spanish for Christ Child, due to its annual occurrence off the South American coast around Christmas, and may also be sensitive to anthropogenic influence (see Figure 6). The Southern Oscillation occurs as a 2–5-year periodic reversal in the east-west pressure gradient associated with the present equatorial wind circulation, called Walker circulation, across the Pacific leading to a reversal in wind direction and changes in temperature and precipitation. The easterly wind in the West Pacific becomes a westerly. As a consequence, the strong trade winds are weakened, affecting climate globally (e.g., crop failures in Australia, flooding in the USA, and the Indian monsoon in India). The Southern Oscillation in turn leads to large-scale oceanic fluctuations in the circulation of the Pacific Ocean and sea-surface temperatures, which is called El Niño. The interannual variability, although, is not yet fully understood; consideration of a wider range of tropical and extratropical influences is needed. A counterpart to the ENSO in the Pacific is the North Atlantic Oscillation (NAO), which is essentially an oscillation in the pressure difference across the North Atlantic and is described further in Figure 7.

Monsoons

The monsoons (derived from Arabic, *mauism*, meaning season or shift in wind) are seasonally reversing winds and one of the most pertinent features of the global atmospheric circulation. The best-known examples are the monsoons over the Indian Ocean and, to some extent, the western Pacific Ocean (tropical region of Australia), the western coast of Africa, and the Carribbean. Monsoons are characteristic for wet summer and dry winter seasons, associated with strong winds and cyclone formation. They occur due to differing thermal characteristics of the land and sea surfaces. Land having a much smaller heat capacity than the ocean, emits heat from solar radiation more easily, leading to upward heat (cumulus) convection. In the summer season, this leads to a pressure gradient and thus wind from the land to the ocean in the upper layers of the atmosphere and subsequent conserving flow of moisture-rich air from the sea back inland at lower levels. This leads to monsoonal rains, increased latent heat release, and intensified monsoon circulation. During the monsoons of the winter season, the opposite of the summer season monsoon takes place, although less pronounced, since the thermal gradient between the land and sea is reversed. The winter monsoons thus lead to precipitation over the sea and cool dry land surfaces.

ANDREAS A. AIGNER AND KLAUS FRAEDRICH

See also **Fluid dynamics; General circulation models of the atmosphere; Hurricanes and tornadoes; Lorenz equations; Navier–Stokes equation**

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ATTRACTOR NEURAL NETWORKS

Neural networks with feedback can have complex dynamics; their outputs are not related in a simple way to their inputs. Nevertheless, they can perform computations by converging to attractors of their dynamics. Here, we analyze how this is done for a simple example problem: associative memory, following the treatment by Hopfield (1984) (see also Hertz, et al., 1991, Chapters 2 and 3).

Let us assume that input data are fed into the network by setting the initial values of the units that make it up (or a subset of them). The network dynamics then lead to successive changes in these values. Eventually, the network will settle down into an attractor, after which the values of the units (or some subset of them) give the output of the computation. The associative memory problem can be described in the following way: there is a set of p patterns to be stored. Given, as input, a pattern that is a corrupted version of one of these, the attractor should be a fixed point as close as possible to the corresponding uncorrupted pattern.

We focus on networks described by systems of differential equations such as

$$\tau_i \frac{\mathrm{d}u_i}{\mathrm{d}t} + u_i(t) = \sum_{j \neq i} w_{ij} g[u_j(t)]. \qquad (1)$$

Here, $u_i(t)$ is the net input to unit i at time t and $g(\,)$ is a sigmoidal activation function ($g' > 0$), so that $V_i = g(u_i)$ is the value (output) of unit i. The connection weight to unit i from unit j is denoted w_{ij}, and τ_i is the relaxation time. We can also consider discrete-time systems governed by

$$V_i(t+1) = g\left[\sum_j w_{ij} V_j(t)\right]. \qquad (2)$$

Here, it is understood that all units are updated simultaneously. In either case, the "program" of such a network is its connection weights w_{ij}.

In general, three kinds of attractors are possible: fixed point, limit cycle, and strange attractor. There are conditions under which the attractors will always be fixed points. For nets described by the continuous dynamics of Equation (1), a sufficient (but not necessary) condition is that the connection weights be symmetric: $w_{ij} = w_{ji}$. General results about the stability of recurrent nets were proved by Cohen & Grossberg (1983). They showed, for dynamics (1), that there is a Lyapunov function, that is, a function of the state variables u_i, which always decreases under the dynamics, except for special values of the u_i at which it does not change. These values are fixed points. For values of the u_i close to such a point, the system will evolve either toward it (an attractor) or away