A Mass Budget of an Ensemble of Transient Cumulus Clouds Determined from Direct Cloud Observations

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ABSTRACT

A parametric model of an individual transient cumulus cloud is derived which allows the incorporation of direct cloud observations. An ensemble of these transient cumulus clouds is constructed using empirical cloud population statistics. It is shown that the mass budget (vertical mass flux, lateral detrainment and final detrainment) of this cloud ensemble can be quantitatively determined by direct cloud observations (number of clouds, cloud cover) without additional aerological information.

1. Introduction

The interaction of cumulus convection and the large-scale environment is a central problem of current research. Different approaches give insight into the mechanisms (lateral mixing, compensating subsidence) by which an ensemble of cumulus clouds modifies the large-scale moisture, temperature and vorticity fields (Kuo, 1965; Ooyama, 1971; Arakawa and Schubert, 1974; Fraedrich, 1974). These mechanisms of interaction depend on the cloud model used by the parameterization theory. Ooyama (1971) assumes a cloud ensemble to be represented by one-dimensional entraining and detraining bubbles of different sizes. Arakawa and Schubert (1974) assume the cloud ensemble to be composed of one-dimensional steady-state plumes of different sizes with entrainment at the lateral boundaries of the cloud and final detrainment. Kuo (1965) assumes lateral mixing of undiluted "hot towers." These parametric schemes are tested by interpreting large-scale diagnostic budgets with the fluxes produced by an ensemble of individual model clouds.

The main purpose of this paper is the development of a model of an individual transient cumulus cloud (in comparison with the steady state plume) which allows the incorporation of direct observations; and the quantitative determination of the mass budget of an ensemble of transient cumulus clouds obtained from cloud population statistics using direct observations (in comparison with results from budget studies).

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2. Individual clouds

The budget of a conserved quantity I' of an individual one-dimensional cumulus cloud is obtained by integration of the Eulerian operator $dI'/dt=0$ over the horizontal cloud area a using the equation of mass continuity and the Leibniz rule of differentiation (see e.g. Fraedrich, 1974):[2]

$$\frac{\partial \rho a I}{\partial t}+\frac{\partial m I}{\partial z}=I_R\left(\frac{\partial \rho a}{\partial t}+\frac{\partial m}{\partial z}\right), \tag{1}$$

with the area-integrated vertical mass flux m and the area-averaged conserved quantity I given by

$$m=\int_0^a \rho w df, \quad I=a^{-1}\int_0^a I' df.$$

Thus the number of independent variables has been reduced to time and vertical coordinate, whereas the dependent cloud variables are represented by area averages.

The vertical mass flux at the boundary of the cloud area has been omitted in (1). The left-hand side represents the storage and vertical flux divergence of the conservative quantity I. The right-hand side can be physically interpreted by

$$\text{Entrainment } e\text{:} \quad \left(\frac{\partial \rho a}{\partial t}+\frac{\partial m}{\partial z}\right)>0, \quad \text{with} \quad I_R=I_e$$

$$\text{Detrainment } d\text{:} \quad \left(\frac{\partial \rho a}{\partial t}+\frac{\partial m}{\partial z}\right)<0, \quad \text{with} \quad I_R=I.$$

[2] A list of symbols is given in an appendix.

Thus the entrainment (detrainment) appears (i) as the vertical convergence (divergence) of the cloud mass flux; (ii) as the horizontal expansion (lateral mixing) of the rising cloud; or (iii) as the combination of both. Accordingly, two fundamentally different simple cloud models can be defined depending on the interpretation of the entrainment or detrainment processes.

a. A steady-state cloud model

This cloud model is obtained from (1) for steady-state conditions and is usually described as an entraining jet with final detrainment only at the maximum cloud depth. Thus the vertical variation of the cloud variable I depends on the entrainment rate and on the environmental distribution I_e of the conserved quantity:

$$\frac{\partial I}{\partial z}=\lambda(I_e-I). \tag{2}$$

The entrainment rate λ in (2) is defined as

$$\lambda=\frac{1}{m}\frac{\partial m}{\partial z}, \tag{3}$$

which leads to an exponentially increasing mass flux distribution, if λ is assumed to be independent of height:

$$m=m_0 \exp\lambda z, \tag{4}$$

where m_0 is the mass flux at cloud base. Entrainment of mass is given by

$$e=\frac{\partial m}{\partial z}=\lambda m_0 \exp\lambda z, \tag{5}$$

while detrainment occurs only in an infinitesimally thin layer δz at the cloud top z_t as a final detrainment:

$$d_f=m(z_t)\delta z^{-1}. \tag{6}$$

This entraining jet model has been incorporated into various parametric schemes (Arakawa and Schubert, 1974; Ogura and Cho, 1973; etc.).

b. A transient cloud model with a vertically constant mass flux

This model describes a cloud with a growing and a decay period. The growing period is characterized by a vertically constant cloud mass flux so that mass entrainment and detrainment balance. The detrainment of cloud properties (heat, moisture, liquid water, vorticity, etc.) can be interpreted as a storage term which leads to the formation of the visible cloud whereas the rising cloud mass defines the active cumulus convection process. When there is no longer rising mass the cloud has reached maturity and the decay phase starts. During the decay phase the detrained cloud properties are laterally mixed into the large-scale environment, i.e., the stored cloud substance (visible cloud) is eroded into the large-scale environment. A cloud process similar to the above has been described by Malkus and Scorer (1955) and others. Thus the growing period of the transient cloud can be described by

$$\frac{\partial \rho a I}{\partial t}+m_0\frac{\partial I}{\partial z}=I_e\frac{\partial \rho a}{\partial t}, \tag{7}$$

where m_0 is vertically constant but still time-dependent.

Eq. (7) can be integrated over the growing period from $t=0$, where the cloud area is zero, to the state of maturity after the half-lifetime $t=\Delta t$ with the maximum cloud area

$$a=\int_0^{\Delta t} \partial a/\partial t\, dt.$$

Additionally, the integral of the flux divergence of the cloud property I [the second term of Eq. (7)] is simplified by a (time related) top hat profile:

$$\int_0^{\Delta t} m_0\frac{\partial I}{\partial z}dt=\left(m_{c0}\frac{\partial I_c}{\partial z}\right)\Delta t,$$

i.e., after integration over the growing stage the mass flux and the vertical gradient of the cloud property are represented by their characteristic values m_c and $\partial I_c/\partial z$. Thus (7) yields

$$m_{c0}\frac{\partial I_c}{\partial z}=\frac{\rho a}{\Delta t}(I_e-I_c). \tag{8}$$

This shows that (in an integrated sense) the storage of cloud properties due to detrainment during the growing period (right-hand side) is balanced by the vertical flux divergence of the cloud property I_c. It is possible to introduce a new entrainment factor λ^* which is the same as a detrainment factor of an entraining and detraining rising parcel. This allows a simpler interpretation of the transient cloud model from the Lagrangian point of view, in terms of a parcel rising from cloud base to cloud top. During the ascent which takes the time of the growing period, the parcel entrains environmental properties and detrains parcel properties at the same rate. The parcel properties are left behind the rising parcel and form the visible cloud, i.e., they are locally stored. After the parcel has reached its final depth, or the cloud has reached the state of maturity, the visible cloud, i.e., the stored or detrained cloud substance, is laterally mixed into the large-scale environment. This kind of model can be further extended to a series of parcels rising through air detrained from former ones.

Additionally, complex microphysics may be included. However, it is not the purpose of this paper to complicate the matter but to introduce first and simply applicable principles, i.e., to use quantities which are directly observable.

Thus Eq. (8) can be written in a form which is similar to the entraining jet model:

$$\frac{\partial I_c}{\partial z}=\lambda^*(I_c-I_e), \tag{9}$$

where the new entrainment factor λ^* has been defined as

$$\lambda^*=\frac{1}{m_{c0}}\frac{\rho a}{\Delta t}. \tag{10}$$

Consequently, the lateral entrainment and the lateral detrainment of mass is given by

$$d_l=-e=\lambda^* m_{c0}=\frac{\rho a}{\Delta t}, \tag{11}$$

which takes place between cloud base ($z=0$) and cloud top (z_t) at the same rate. The final detrainment occurs at the cloud top only in an infinitesimally thin layer δz, i.e.,

$$d_f=m_{c0}\delta z^{-1}. \tag{12}$$

It is immediately obvious that the right-hand side of (11) is a directly observable quantity, in contrast to parameters of the entraining jet model. A statistical radar analysis of convective processes in the tropics [Venezuela: Cruz, 1973; Betts and Stevens, 1974] and in middle latitudes [Germany: Singler, 1975] shows that there is a good correlation between the lifetime $2\Delta t$ and the maximum cloud area a, suggesting the relation

$$\lambda^* = m_{c0}=\frac{\rho a}{\Delta t}=c_1, \tag{13}$$

where c_1 is an empirical constant.

The constant c_1 is taken to be the same for all clouds of a given type as it is indicated by the observations of (tropical) mesoscale cumulonimbus systems and for (middle latitude) individual cumulonimbus towers. Actually, this constant c_1 may be different for the classes of (nonprecipitating) trade wind cumuli and the (precipitating) cumulonimbus towers, but in the following it will be assumed to be the same for both classes of convection (this assumption does not lead to a change in the following formulas). From Eq. (13) and the above-mentioned observations one obtains a relationship between the cloud mass flux m_{c0} and the entrainment factor λ^* such that a large cloud mass flux is related to a small entrainment factor and consequently to deep cumulus clouds (and vice versa). According to this transient cloud model and its meteorological interpretation it is only the magnitude of the mass flux m_{c0} at cloud base which defines the entrainment factor λ^* for the total cloud process and consequently the vertical cloud depth, its area and lifetime.

This transient cloud model has also been included into parametric schemes (Fraedrich, 1973, 1974; see also Kuo, 1965, 1974). If the conservative quantity I is specified by the total energy and (moist) potential vorticity, one obtains the vertical distribution of energy and vorticity within the cloud. The temperature, specific humidity and liquid water content can easily be calculated after parameterizing the rainout process.

3. Empirical relationships between cloud depth, radius and entrainment factor

In order to apply the transient cloud model to direct observations of a cloud population some additional relationships which are experimentally verified will be introduced:

1) A relation between the entrainment factor and the cloud radius to give a radius-dependent mass flux.

2) A relation between the cloud radius and cloud depth to define the layer within which the mass flux of an individual transient cloud is known.

Using these relationships one is able to derive the mass budget of a transient cloud ensemble composed of individual clouds with different radius (and depth) given the cloud number density depending on the cloud radius. The latter will be described in the following section.

For rising thermals a relationship between the entrainment factor λ^* and cloud radius r is well established (e.g. Simpson *et al.*, 1965) and takes the form

$$\lambda^*=\frac{3\alpha}{r}, \tag{14}$$

where 3α is an empirical constant.

In addition, it has been suggested from observations that there is a linear relationship between cloud radius r and depth z_t (Plank, 1969; Betts, 1973):

$$\frac{r}{z_t}=\gamma, \tag{15}$$

where γ is an empirical constant.

For undisturbed (e.g., non-precipitating trade wind cumuli) situations the individual clouds are characterized by suppressed growth. It appears that on the average they are as tall vertically as they are wide horizontally (Plank, 1969; Betts, 1973), i.e., $r/z_t=\gamma \approx 0.5$. Disturbed situations (e.g., precipitating cumulonimbus towers) show a large natural cloud growth and the height-radius ratio can be approximated by $r/z_t=\gamma\approx 0.2$ (Betts, 1973).

The dependence of the transient cloud parameters (vertical mass flux, lateral and final detrainment) on the entrainment factor can now be transformed to cloud depth or width.

4. Statistical cloud population parameters

Before the model of an individual transient cloud (Section 1) and the empirical relationships (Section 2) can be applied to direct cloud observations some additional information on cloud population statistics are necessary and will be presented in the following.

A cloud ensemble within a large-scale unit area F is composed of a number of individual cumulus clouds. A relation between the number of clouds and their size can be described by a distribution function which characterizes the cloud population. An expotential distribution function which is similar to the Marshall-Palmer raindrop-size distribution appears to be applicable to an ensemble of clouds:

$$n(r)=K\exp(-\beta r), \tag{16}$$

where $n(r)$ is the number density, K the effective number density, and β the exponential number density decrease.

Such a distribution has been observationally verified for (non-precipitating) trade wind cumuli over Florida (Plank, 1969) and for the Atlantic ITCZ (Ruprecht *et al.*, 1974). The parameters K, β which define the shape of the cloud number density can be calculated from the total number of clouds and their area cover. The distribution function (16) leads to the accumulated number of clouds:

$$N(r)=-\int_{r_{\max}}^{r} n(r')dr'=N_0 n^*(r), \tag{17}$$

where the order of accumulation starts with the largest cloud radius ($r_{\max}$) observed. The total number of clouds

$$N_0=\frac{K}{\beta} \tag{18}$$

is given for the radius interval $r\rightarrow r_{\min}=0$ and $r_{\max}=\infty$ with all clouds included. This can be confirmed by observations (Plank, 1969) because the cloud radii cover a wide spectrum if the cloud population and the large-scale unit area are large enough. According to these observations the profile function can be approximated by

$$n^*(r)=[\exp(-\beta r)]_{r_{\max}}^{r}\approx\exp(-\beta r) \tag{19}$$

because $\exp(-\beta r_{\max})\approx 0$ (and keeping in mind that the accumulation increases with decreasing radius).

The accumulated area covered by an ensemble or circular clouds ($a=\pi r^2$) is given by

$$A(r)=\int_{r}^{r_{\max}} \pi r'^2 n(r')dr'=A_0 a^*(r), \tag{20}$$

with the total area covered by the cloud ensemble

$$A_0=\frac{K}{\beta^3}2\pi. \tag{21}$$

The profile function

$$a^*(r)\approx\left[\frac{(\beta r)^2}{2}+\beta r+1\right]\exp(-\beta r) \tag{22}$$

also approaches the value $a^*\rightarrow 1$ for $r\rightarrow r_{\min}$. As the fractional cloud cover

$$\sigma=\frac{A_0}{F} \tag{23}$$

and the total number of clouds N_0 in (18) are directly observable quantities, the parameters K, β of the cloud number density function can simply be determined from (18) and (21), i.e.,

$$K^2=2\pi\frac{N_0^3}{A_0},\quad \beta^2=2\pi\frac{N_0}{A_0}. \tag{24}$$

The factor

$$\frac{K}{\beta^2}=\frac{N_0}{\beta}=\left(\frac{N_0 A_0}{2\pi}\right)^{\frac{1}{2}}$$

will be used later.

Now, the accumulated mass flux of an ensemble of transient cumulus clouds can be derived using the empirical cloud population statistics (16), the entrainment factor-radius dependence (14), and the parametric model describing an individual transient cumulus cloud (13):

$$M(r)=\sum_{i=1}^{N(r)} m_{c,i}=\int_{r_{\max}}^{r} n(r')m(r')dr'=M_0 m^*(r), \tag{25}$$

where the total mass flux

$$M_0=\frac{K}{\beta^2}\frac{c_1}{3\alpha}=\left(\frac{N_0 A_0}{2\pi}\right)^{\frac{1}{2}}\frac{c_1}{3\alpha} \tag{26}$$

has to be physically interpreted as the ensemble mass flux at cloud base as all the clouds included are assumed to have the same cloud base. The profile function is given (and well-approximated) by

$$\begin{aligned} m^*(r)&=(\beta r+1)\exp(-\beta r)-(\beta r_{\max}+1)\exp(-\beta r_{\max}) \\ &\approx(\beta r+1)\exp(-\beta r). \end{aligned} \tag{27}$$

5. Mass flux and final and lateral detrainment of an ensemble of transient clouds

The cloud width-depth ratio γ allows a direct transformation of the radius-dependent accumulated cloud frequency $N(r)$, area $A(r)$, and mass flux $M(r)$ distribution into height dependence. Using (15) this transformation is directly applicable with z instead of z_t since the number, area, and mass flux of each individual transient cloud are height-independent. This leads to the accumulated frequency of cloud top heights from (15), (17) and (19):

$$N(z)=N_0\exp(-\beta\gamma z),$$

where the term $\exp(-\beta r_{\max})$ has been omitted for simplicity [and according to Eq. (19)]; in addition, we also obtain the vertical distribution of cloud cover $A(z)$ and the vertical distribution of the cloud mass flux $M(z)$.

The vertical mass flux profile of the cloud ensemble is obtained by combining (15), (17), (19) and (25):

$$M(z)=M_0(\beta\gamma z+1)\exp(-\beta\gamma z), \tag{28}$$

with the vertical mass flux M_0 at cloud base given by (26). The mass continuity equation leads to the final detrainment of all individual clouds within the cloud ensemble:

$$D_f=-\frac{\partial M}{\partial z}=D_{f0}(\beta\gamma z)\exp(-\beta\gamma z), \tag{29}$$

where

$$D_{f0}=M_0\beta\gamma=N_0c_1\frac{\gamma}{3\alpha}.$$

Transient clouds also detrain (and entrain at the same rate) in the layer between cloud base and top as given by Eq. (11); i.e., the lateral detrainment D_l of all clouds is proportional to the accumulated distribution of cloud top heights $N(z)$:

$$D_l=\sum_{i=1}^{N(z)} d_{l,i}=c_1N(z)=D_{l0}\exp(-\beta\gamma z), \tag{30}$$

with

$$D_{l0}=c_1N_0.$$

The constant of proportionality c_1 is defined by Eq. (13). This is due to the physical interpretation of the transient cloud model where the production rate of cloud substance (and its erosion rate) is assumed to be the same for all clouds which belong to the class of precipitating or non-precipitating cumuli.

A summary of the mass budget of the ensemble of transient clouds shows the following: The vertical mass flux M_0 entering the cloud base is balanced by the final vertically integrated detrainment from all cloud tops, i.e.,

$$M_0=\int_0^\infty D_f dz=\left(\frac{N_0A_0}{2\pi}\right)^{\frac{1}{2}}\frac{c_1}{3\alpha}. \tag{31}$$

Lateral detrainment is individually balanced by lateral entrainment. Integration of the lateral detrainment (30) over the total cloud layer yields

$$\int_0^\infty D_l dz=\left(\frac{N_0A_0}{2\pi}\right)^{\frac{1}{2}}\frac{c_1}{\gamma}. \tag{32}$$

The ratio between vertically integrated final and lateral detrainment is given by the empirical constants defined by the relationships (14) and (15).

The vertical mass flux and the final and lateral detrainment of the cloud ensemble are the main parameters which govern the interaction between large-scale and cloud-scale (see e.g. Ooyama, 1971; Yanai *et al.*, 1973; Fraedrich, 1973, 1974; Kuo, 1965, 1974; Arakawa and Schubert, 1974; etc.). With the transient cloud model defined in Section 1 these parameters can be calculated from direct cloud observations. This cannot be achieved by the entraining jet model (Arakawa and Schubert, 1974) because the mass flux at cloud base remains an unknown variable. An example is presented in the following section.

6. An application

The vertical profiles of the mass flux and the final and lateral detrainment of the cloud ensemble are presented in a normalized coordinate system (Fig. 1) which has to be scaled according to the actual observations. The empirical constants (13) and (14) introduced to described the individual transient cloud are more or less uniquely defined:

$$c_1=10^5\ \text{kg m}^{-1}\ \text{s}^{-1}, \qquad 3\alpha=0.75,$$

where data from Cruz (1973), Betts and Stevens (1974) and Singler (1975) have been used to determine c_1; the entrainment factor 3α is taken for rising thermals from Scorer (1957) and Levine (1965). The other scaling parameters depend on the actual situation observed: the cloud radius-height ratio γ, the total number of clouds N_0 within a prescribed large-scale unit area F, and the fractional cloud cover $\sigma=A_0/F$. As an example we choose a typical undisturbed trade wind situation for which the radius-height ratio yields $\gamma=0.50$; the cloud population parameters are taken from Plank (1969; Florida, 1300–1400 EST, Table 2) with $N_0=513$, $\sigma=0.477$, $F=250$ km^2, so that the cloud area gives $A_0=123$ km^2. This leads to the quantities

$$K\approx 2.6\ \text{m}^{-1}, \qquad \beta\approx 5\times10^{-3}\ \text{m}^{-1},$$

which define the cloud number density function given by (15) and (23), and to the scaling parameters which are related to the large-scale unit area F.

The mass flux M_0/F at cloud base ($z=0$) is given by (26), the amplitude D_{f0}/F of the final detrainment by (29), and the lateral detrainment D_{l_0}/F at cloud

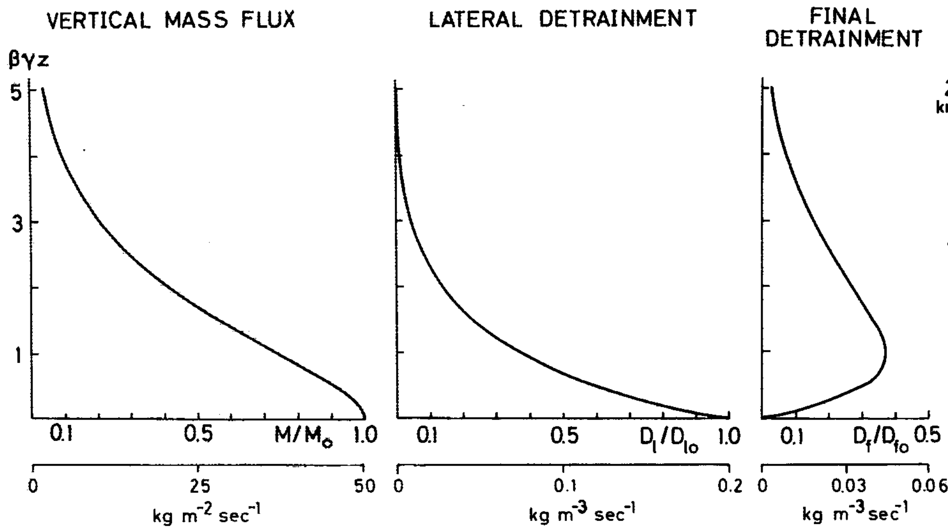

FIG. 1. Vertical distribution of the mass flux, the lateral detrainment (entrainment), and the final detrainment of an ensemble of transient clouds. Right and bottom coordinates are scaled for a specific example (see text).

base by (30); thus we have

$$M_0F^{-1}\approx 50 \text{ kg m}^{-2}\text{ s}^{-1},$$
$$D_{l_0}F^{-1}\approx 0.2 \text{ kg m}^{-3}\text{ s}^{-1}$$
$$D_{f_0}F^{-1}\approx 0.125 \text{ kg m}^{-3}\text{ s}^{-1}.$$

The vertical axis is scaled by $\beta\gamma=2.5$ km^{-1} (Section 4) so that the actual height above cloud base can be deduced (Fig. 1). Using these data one can derive the vertical distribution of the mass flux, the final detrainment, and the lateral detrainment (entrainment). The latter is proportional to the accumulated frequency distribution $N(z)$ of cloud heights [see Eq. (30)]. The cloud population model, i.e., the normalized graphs of Eqs. (28–30), are shown in Fig. 1. The quantitative results using the above observations are indicated on the lower and right axes. These results can be compared with diagnostic budget studies of convective conditions. Such studies use spectral cloud ensemble models (Arakawa and Schubert, 1974) to interpret large-scale budget data (Ogura and Cho, 1973; Nitta, 1975), whereas in this study the cloud population parameters are derived from direct observations. It appears that for undisturbed situations the profiles of the cloud ensemble mass fluxes are similar. However, the mass flux of the transient cloud ensemble is larger due to the small area F which is nearly 50% cloud covered. This directly observed cloud cover should not be interpreted as the relatively small area (from a few up to 10%) which is covered by active updrafts within the cumulus clouds, an assumption which many parametric models are based upon. Instead it is the area coverage of the total cloud process (with the inactive stages due to detrainment or storage of cloud property being included in the transient cloud model). In addition the distribution of the total (lateral plus final) detrainment shows acceptable agreement. The mass flux and the detrainment of the cloud ensemble are the parameters which determine the input of the convection into the large scale. Spectral cloud-ensemble models with steady-state plumes representing an individual cloud do not permit us to distinguish between lateral and final detrainment.

7. Concluding remarks

This paper introduces a first and simplified step toward an application of direct cloud observations (e.g., by satellite) to the parameterization of cumulus convection, i.e., to determine the input from the cumulus scale into the large scale and vice versa. With the attempt described here it may be possible to directly deduce the large-scale sources of dynamic and thermodynamic quantities due to convective activity which are described by the processes of "compensating subsidence" and "lateral mixing." Thus, knowing the large-scale distribution of latent and sensible heat, vorticity, the conditions at cloud base, and direct cloud observations (cloud number, cloud cover), one is able to determine the large-scale sources of heat, moisture and vorticity due to cumulus convection (e.g., Fraedrich, 1974). However, it should be noted that a numerical simulation of a cloud ensemble model is also needed to prove the observationally verified distribution function of clouds and the empirical constants introduced. These constants indicate the structure of the individual clouds embedded in a cloud ensemble and lead to such additional relationships as: (i) the depth-width ratio γ of the cloud height z_t as a function of the radius r [which appears to be different for precipitating and non-precipitating clouds]; (ii) the cloud production rate c_1 as the ratio of maximum cloud area a and lifetime Δt [which may also be different for raining and non-raining clouds (but is assumed to be the same for the example presented)]; and (iii) the constant of proportionality 3α relating the entrainment factor λ^* to the inverse of the cloud radius r^{-1}. In addition, the model of an individual transient cloud establishes a simple relation between the cloud mass flux m_{c0} and its entrainment factor λ^* by introducing the constant $c_1=\lambda^* m_{c0}$.

Now, given the cloud number density depending on the cloud radius, one is able to describe a cloud population by the following distribution functions: the accumulated cloud number and cloud area cover, and the accumulated cloud mass flux (including lateral and final detrainment) using the relationships between radius and entrainment factor (constant 3α), and between entrainment factor and mass flux (constant c_1). These radius-dependent distribution functions can also be transformed to height-dependent functions by using the cloud radius to height ratio (constant γ), thus making possible a better physical interpretation.

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APPENDIX

List of Symbols

t, z, r	time, height above cloud base, cloud radius
ρ, w	density, vertical velocity
$I', (I)$	conserved cloud property (area averaged)
a, A	area; individual cloud, cloud ensemble
m, M	vertical mass flux; individual cloud, cloud ensemble
d, D	mass detrainment; individual cloud, cloud ensemble (with subscripts f, l: final, lateral detrainment)
$n; K, \beta$	cloud number density; related coefficients
N	accumulated number of clouds
α, γ, c_1	empirical constants
n^*, m^*, a^*	vertical profiles of accumulated cloud number, mass flux, and area of the cloud ensemble
λ, λ^*	entrainment factor; plume, thermal
σ	fractional cloud cover
F	large-scale unit area

Subscripts

$0, t$	cloud base, cloud top
e	environment
c	transient cloud
R	value at the boundary of the cloud area
i	summation-index

REFERENCES

Arakawa, A., and W. Schubert, 1974: Interaction of a cumulus cloud ensemble with the large-scale environment, Part I. *J. Atmos. Sci.*, **31**, 674–701.

Betts, A. K., 1973: A relationship between stratification, cloud depth, and permitted cloud radii. *J. Appl. Meteor.*, **12**, 890–893.

——, and M. A. Stevens, 1974: Rainfall and radar echo statistics, Venezuelan International Meteorological and Hydrological Experiment, 1972. Atmos. Sci. Paper, Colorado State University [US ISSN 0067-0340] 151 pp.

Cruz, L., 1973: Venezuelan rainstorms as seen by radar. *J. Appl. Meteor.*, **12**, 119–126.

Fraedrich, K., 1973: On the parameterization of cumulus convection by lateral mixing and compensating subsidence, Part I. *J. Atmos. Sci.*, **30**, 408–413.

——, 1974: Dynamic and thermodynamic aspects of the parameterization of cumulus convection, Part II. *J. Atmos. Sci.*, **31**, 1838–1849.

Kuo, H. L., 1965: On the formation and intensification of tropical cyclones through latent heat release by cumulus convection. *J. Atmos. Sci.*, **22**, 40–63.

——, 1974: Further studies of the parameterization of the influence of cumulus convection on large-scale flow. *J. Atmos. Sci.*, **31**, 1232–1240.

Levine, J., 1965: The dynamics of cumulus convection in the trades—A combined observational and theoretical study. Ref. No. 65-43, Woods Hole Oceanographic Institution, 129 pp.

Malkus, J. S., and R. J. Scorer, 1955: The erosion of cumulus towers. *J. Meteor.*, **12**, 43–57.

Nitta, T., 1975: Observational determination of cloud mass flux distributions. *J. Atmos. Sci.*, **32**, 73–87.

Ogura, Y., and H. R. Cho, 1973: Diagnostic determination of cumulus cloud populations from large scale variables. *J. Atmos. Sci.*, **30**, 1276–1286.

Ooyama, K., 1971: A theory on parameterization of cumulus convection. *J. Meteor. Soc. Japan*, **49** (Special Issue), 744–756.

Plank, V. G., 1969: The size distribution of cumulus clouds in representative Florida populations. *J. Appl. Meteor.*, **8**, 46–67.

Ruprecht, E., L. Breuer, K. Bucher, E. Scheidtmann and W. Zäck, 1974: Radarbeobachtungen der Konvektionsgebiete im Bereich der innertropischen Konvergenzzone über dem äquatorialen Atlantischen Ozean. *"Meteor"-Forsch.-Ergebnisse*, Reihe B, Vol. 11, 11–23.

Scorer, R. J., 1957: Experiment on convection of isolate masses of buoyant fluid. *J. Fluid Mech.*, **2**, 583–594.

Simpson, J., R. H. Simpson, D. A. Andrews and M. A. Eaton, 1965: Experimental cumulus dynamics. *Rev. Geophys.*, **3**, 387–431.

Singler, A., 1975: Wachstumsgeschwindigkeit und Lebensdauer konvektiver Niederschlagswolken. *Arch. Meteor. Geophys. Bioklim.*, **A24**, 33–39.

Yanai, M., S. Esbensen and I. H. Chu, 1973: Determination of bulk properties of tropical clusters from large scale heat and moisture budgets. *J. Atmos. Sci.*, **30**, 611–627.