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Application of a Markov technique to the operational, short-term forecasting of rainfall

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The short-term prediction of precipitation remains a major forecasting problem. Most Australian Regional Forecasting Centres (RFC) still use basically subjective techniques and exhibit modest levels of skill.

In this study it is shown that the Markov chain technique has sufficient skill to form an objective basis for the short-term operational prediction of the probability of precipitation. This is demonstrated by the performance of the technique in two comparative trials. The first trial was a real-time trial for Melbourne, carried out over a period of more than two years. In this trial the performance of the Markov chain model, as judged by the standard measurement of skill (half-Brier scores), was clearly superior to probability of precipitation forecasts issued by the Victorian Regional Office. The second trial involved comparing the Markov chain technique with persistence and climatology forecasts for all eight Australian RFCs, using 14 years of data for verification. Again it was found that the Markov chain technique showed such a high level of skill relative to persistence and climatology that it can be considered for use as an operational technique for short-term rainfall prediction. Finally, we discuss how the skill level of the Markov chain technique can be enhanced by combining it in a linear manner with other independent forecasting techniques, such as numerical weather prediction, as part of an integrated system.

Introduction

There are currently three methods (not necessarily independent) used by the Australian Bureau of Meteorology for the prediction of precipitation. These are: subjective forecasts by duty forecasters; deterministic forecasts obtained from numerical weather prediction models; and statistical forecasts. The last two methods fall into the class of objective techniques. Predictions from both deterministic and statistical methods are available to the duty forecaster and are part of the information used in formulating the subjective predictions.

Subjective forecasts still remain the official Bureau of Meteorology forecasts and are provided by the RFCs. They take two main forms: worded forecasts issued directly to the public, and quantitative precipitation forecasts in discrete ranges. The quantitative forecasts are not given directly to the public, but reflect the worded forecasts and form the basis for verifying the precipitation forecasts.

The deterministic forecasts of precipitation for the Australian region are obtained from the operational numerical weather prediction (NWP) model, FINEST, which is run twice per day (based on 1100 UTC and 2300 UTC data). These 36-hour forecasts are quantitative and amounts predicted at each grid-point are contour-plotted and distributed to the RFCs by the National Meteorological Centre (NMC).

The statistical predictions of precipitation at present are Model Output Statistics (MOS) forecasts of rainfall amount, based on regressions between observed rainfall and forecasts of various parameters produced by FINEST. The MOS forecasts also are distributed by NMC, but unlike the US MOS forecasts, are not issued as probabilities.

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During the past twenty years the level of skill of short-term forecasts (up to 24 hours) of synoptic-scale fields prepared for the Bureau of Meteorology by NMC has improved dramatically. For example, the standard measure of skill, the S_1 skill score, for 24-hour Australian region NWP forecasts of mean sea level pressure (MSLP) has been reduced from the low 50s to the mid 30s (see Fig. 1). The improvement of FINEST forecasts over subjective (manual) forecasts of MSLP is now greater than five skill points. However, the level of skill in forecasting precipitation has not shown such a level of improvement and subjective forecasts remain the official Bureau of Meteorology product. As an illustration, the percentage of correct forecasts for the Victorian Regional Office (VRO) of measureable rainfall (amounts equal to or greater than 0.1 mm in the Melbourne rain-gauge) are shown in Fig. 2(a) for the years 1982-1987. The forecasts were issued at 0600 hours local time and had a period of validity of 18 hours. The mean annual percentage of correct forecasts are seen to lie in a band approximately between 40 to 60 per cent correct. It is seen also, in Fig. 2(b), that there is a strong tendency to overforecast rainfall occurrence with the annual values of bias consistently lying between +30 and +90 per cent. Although there are some fluctuations, these results do not seem to indicate a significant improvement of the forecasts with time. Similar percentages of correct scores and bias values are recorded at most other RFCs in Australia, with one notable exception, Adelaide, for which the forecasts do appear to improve with time as shown in Figs 3(a) and (b). It should be pointed out that the percentage of correct forecasts depends on the sample relative frequency, so that higher percentages of correct forecasts in Adelaide do not necessarily indicate higher levels of skill. Rainfall verification statistics for all RFCs are available from the authors and are currently being prepared for publication by the Bureau of Meteorology.

Fig. 1 The improvement in MSLP forecasting with time as measured by the S_1 skill score. The straight line indicates the 12-month running mean.

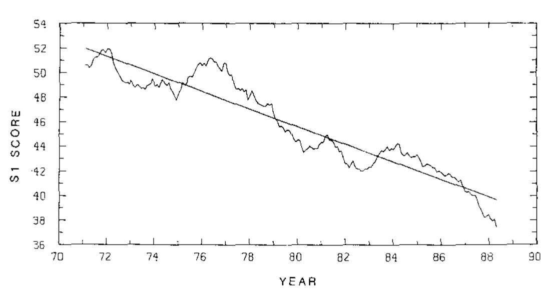


Fig. 2 (a) Percentage of correct rain forecasts for Melbourne for the period 1982-87. Percentage of forecasts correct = number of correctly forecast raindays/total number of forecast raindays. (Perfect score = 100%.) (b) Forecast bias for Melbourne for the same period. Forecast bias = [(total number of forecast raindays/total number of observed raindays) − 1]. (Perfect score = 0%.)

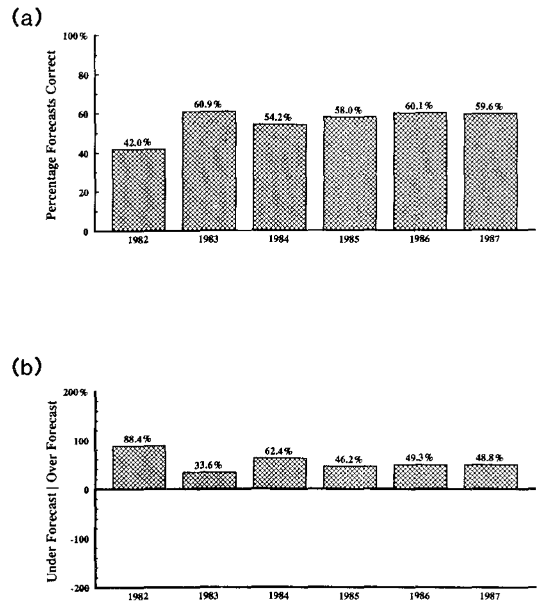
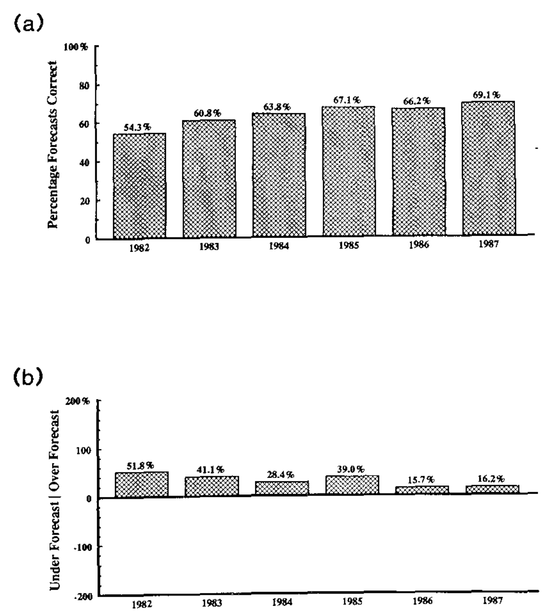


Fig. 3 Same as Fig. 2, except for Adelaide.



Recently, an alternative statistical approach was tested in an attempt to improve the skill of short-term rainfall prediction, particularly over the first 12 hours. The technique used was the Markov chain model in which the probability of precipitation being recorded at a given station is assumed to depend only on the present and immediately preceding weather state defined from routine surface observations (Fraedrich and Muller 1983; Miller and Leslie 1985).

Two major real-time trials of methods for predicting precipitation for the 12-hour period 6 am to 6 pm were carried out. These studies were for the RFCs at Melbourne and Darwin for the winter season of 1986 and the summer season of 1986/87, respectively (Fraedrich and Leslie 1987, 1988). In both of these trials a variety of techniques for predicting precipitation were evaluated and compared, and it was found that in each trial the best scheme was the Markov chain technique. It was also found that the Markov method could be improved significantly by linearly combining the results with those from other techniques, such as rainfall predictions from the NWP model, FINEST.

Statistical models, such as the Markov model, produce probability forecasts as their output. These probability forecasts have an important advantage over the currently used worded forecasts, e.g. mainly fine, isolated showers, etc., because they quantify the degree of certainty present in the forecasts. This quantification is helpful to users in making decisions which depend on the weather. Issuing probability forecasts also reflects the true character of the forecasting process, which is not purely deterministic. At international meteorological centres numerical weather predictions are beginning to be issued along with estimates of confidence in the forecast, or are statistically corrected because of errors in the initial conditions, deficiences in the model formulation, and the essentially stochastic nature of turbulent flow. For these reasons this paper will concentrate on evaulation of probability forecasts of precipitation. We note that in Australia probabilistic forecasts of precipitation currently are available in real-time in Melbourne and Canberra (Stern 1980; Dahni et al. 1984; Mason 1982), but are issued officially to the public only in Canberra.

The present study has a two-fold purpose. First, a Markov chain technique, which is an improved version of that used by Fraedrich and Leslie (1987, 1988), is introduced. The new version incorporates a large number of additional covariates such as the north-south wind component, the change in low-level cloud amount and the middle-level cloud amount. Second, two investigations are made of the skill of the techniques in 12-hour probability of precipitation forecasting. In the first investigation the Markov chain model forecasts are compared directly with the probability forecasts produced by the VRO duty forecasters in real-time over the 25-month period from the end of May 1986 to the end of June 1988. It is important to note that the VRO duty forecasters have not been trained in making probability of precipitation forecasts and this may have implications for their level of skill. In the second investigation, the Markov chain technique is applied to all eight RFCs and evaluated against persistence and climatological forecasts in an attempt to see if the skill levels are sufficiently high to justify operational implementation by the NMC. A summary of the results of the second investigation is presented here; full details will be published as a separate report by the Bureau of Meteorology Research Centre.

Finally, some comments are made in the discussion and conclusions section on how the ever-increasing range of techniques for forecasting precipitation can best be used.

Methodology

Markov technique

The basis of the statistical forecasting technique to be used in this study is the Markov chain process. A first-order Markov process is one in which knowledge of a parameter at time t, such as a weather state (for example, the current cloudiness or the amount of rain recorded since the last observation time), is sufficient to predict the parameter at some later time (see Essenwanger 1985, pp. 349-359, for a comprehensive treatment of the application of Markov techniques to meteorological problems). A discrete, second-order Markov process uses knowledge of the present state and a previously observed state (at time $t - \Delta t$) to predict future states. High-order Markov processes use progressively more information from past states for their predictions.

By using climatological records it is possible to determine the conditional probabilities that a particular state will occur (for example rain) given various present states. These conditional probabilities are called transition probabilities. In our study we use a hybrid, discrete, second-order Markov technique in which the transition probabilities are approximated by employing multilinear regression over a number of covariates.

Our model is similar to, but more extensive than, the one used by Fraedrich and Leslie (1987, 1988), which in turn was based on the earlier studies by Fraedrich and Muller (1983) and Miller and Leslie (1985). A related, independent model has been developed by Miller (1984).

In our model four mutually exclusive weather states are defined: State 1 is when there is 0-2 octas of total cloud cover and no rain has fallen in the previous 3 hours; State 2 is 3-5 octas of total cloud cover and no rain; State 3 is 6-8 octas of total cloud cover and no rain; and State 4 is when 0.1 mm or more of rain has been recorded at the station in the 3-hour period since the previous observation. The weather states have been chosen to characterise the progression from clear, fine conditions to rainy weather situations in a reasonable manner. No attempt has been made to investigate the effects of choosing different states or more states.

The available three-hourly surface climatic data (which cover the years 1960-1988) have been collected for eight Australian capital cities, and two major studies of the performance of the Markov chain technique have been carried out. The first was a comparison of the Markov method with the probability forecasts of the VRO. For this case Melbourne data from the entire period from 1960 – 1985 were used to develop the Markov model and data from the period 22 May 1986 – 30 June 1988 were used for validation. In the second case data chosen from random years, comprising approximately half of the data for each of the eight stations, were used to develop the models. The remainder of the data were used to verify the models and compare them with persistence and climatology.

The probability that rain will occur is found by performing linear least squares regression on 14 covariates: previous weather state (i, where i has a value of 1-4); the station pressure minus 1000 hPa; the change in station pressure in the last three hours; the dry-bulb temperature minus 18°C; the change in dry-bulb temperature in the last 3 hours; the wet-bulb depression; the change in the wet-bulb depression in the last 3 hours; the east-west component of the wind; the north-south component of the wind; the change in total wind speed in the last 3 hours; the amount of low cloud cover; the change in the amount of low cloud cover in the last 3 hours; the amount of middle cloud cover; and the change in the amount of middle cloud cover. Two additional covariates, sine and cosine harmonics, are employed to represent the annual cycle.

We can write the probability of rain Pr as a function of the month m, the current weather state j, the time of day t, and the forecast period (the number of hours ahead for which the forecast is valid) h:

$$\begin{aligned} Pr\,(j, h, t, m) &= a\,(j, h, t, m) + \sum_{k=1}^{14} b_k\,(j, h)\, X_k \\ &+ b_{15}\,(j, h) \sin(2\pi m/12) \\ &+ b_{16}(j, h) \cos(2\pi m/12) \qquad \ldots 1 \end{aligned}$$

where a (j, h, t, m) is the intercept, $b_k(j, h)$ are the regression coefficients, and X_k are the covariates. There is a different intercept for each starting time, e.g. for 0600, 0900 and 1200 hours.

Linear least squares regression is used to fit the data. With a linear model it is possible to calculate probabilities less than zero or greater than unity. To exclude this possibility we explicitly set negative probabilities equal to zero and probabilities greater than unity equal to unity.

Measure of skill

The accuracy of the probability forecasts is judged by the half-Brier score (Brier 1950), defined as

$$Br = (1/N) \sum_{i=1}^{N} (\delta_i - Pr_i)^2 \qquad \ldots 2$$

where δ_i is the observed probability for the *i*th day ($\delta_i = 1$ if rain occurred within the forecast period; otherwise $\delta_i = 0$), Pr_i the forecast probability for the *i*th day, and N the total number of days. Half-Brier scores can be computed for the Markov model, the VRO predictions, persistence and climatology by substituting the appropriate values of probability into Eqn 2.

The half-Brier score can be interpreted as the mean square error of the probabilistic forecasts. Lower scores indicate more skill; the scores range from zero (perfect score) to unity.

Results

The results of two major investigations of the Markov chain technique for forecasting precipitation are described below.

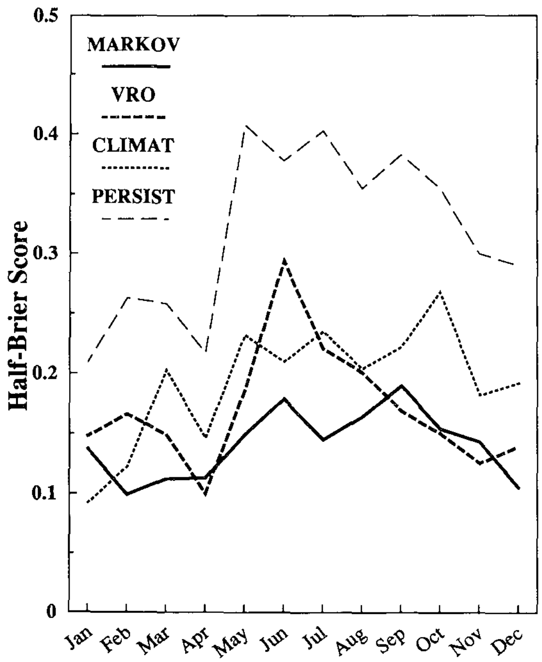
VRO trial

A direct comparison of the probability of precipitation predicted by the Markov chain model and the duty forecasters of the VRO was carried out for a 25-month period. The relative skill in the forecasts is determined by comparing the half-Brier scores. The monthly half-Brier scores for 12-hour forecasts issued at 6 am local time are shown in Fig. 4. Also included are the half-Brier scores for persistence (if it rained yesterday during the forecast period, it will rain today) and for climatology (the observed frequency of rainfall for the years 1960-1985).

The half-Brier score, like the percentage of correct forecasts, depends on sample climate. It also tends to get worse as the sample relative frequency of rain goes from 0 to 0.5, regardless of the skill or calibration of the forecasts (Glahn 1985). However, the comparison shown in Fig. 2 is valid because we are testing different forecast methods for the same station and sample climate.

The Markov model performs very well. It outperforms the VRO, climatology and persistence in most months. There are two exceptions. In January, when there are few days when it rains, climatology is the best guide, and in times of transition, when the weather patterns are highly variable (April, September to November), the VRO forecasts are slightly better than the Markov forecasts, although the standard one-tailed t-test (Miller 1962) shows that these differences are not

Fig. 4 **Monthly half-Brier scores for rain forecasts for the Markov model, the VRO, climatology and persistence for the period 22 May 1986 to 30 June 1988. A perfect forecast gives Br = 0.**



significant at the 5 per cent level. However, for the rest of the year the Markov model performs better than the other schemes. Especially notable is the low monthly variability for the Markov predictions.

The VRO gives the best results for April, but during the winter months the VRO forecasts deteriorate. In June, when there are many rain days, the VRO predictions are not as good as climatology.

Table 1 shows the monthly, seasonal and annual half-Brier scores (means and standard errors). The most reliable comparison is based on the annual values. In this case the Markov model shows an improvement of 19 per cent compared to the VRO forecasts. The improvements over climatology and persistence are 27 and 56 per cent, respectively.

The reliability diagrams for the VRO and Markov model forecasts for this trial are shown in Figs 5(a) and (b), respectively. These diagrams were prepared by dividing up the predicted probability into intervals of 5 per cent and calculating the percentage of occasions on which rain was observed (the relative frequency of rain events) for each prediction interval. The diagonal line indicates a perfect forecast. The VRO results in Fig. 5(a) exhibit a tendency to prefer predictions in tens of per cent rather than values in between. In general the subjective forecasts are biased to predicting probabilities that are too high. The Markov model predictions in Fig. 5(b) are closer to the diagonal line, but for low predicted probabilities are also biased to predicting values that are too high. The Markov model predictions should improve by increasing the length of the validation period (so that the period better reflects the climatological data), but the subjective bias in the VRO forecasts may not change with a longer validation period. The improvement in the Markov model reliability for Melbourne is seen in Fig. 6 when a period of 14 randomly selected years between 1960 and 1988 is used instead of the 25-month period of the VRO trial.

Table 1. Monthly, seasonal and annual half-Brier scores (means and standard errors) for 12-hour rainfall forecasts for Melbourne for the period 22 May 1986 – 30 June 1988.

Period	*VRO*	*CLIM*	*PERSIST*	*MARKOV*
January	0.148±0.037	0.092±0.026	0.210±0.049	0.137±0.014
February	0.166±0.046	0.122±0.030	0.263±0.061	0.099±0.015
March	0.149±0.035	0.203±0.033	0.258±0.056	0.112±0.017
April	0.100±0.029	0.146±0.024	0.217±0.053	0.113±0.017
May	0.186±0.035	0.232±0.019	0.408±0.059	0.149±0.019
June	0.294±0.058	0.210±0.019	0.378±0.052	0.179±0.021
July	0.221±0.040	0.235±0.022	0.403±0.063	0.145±0.024
August	0.201±0.040	0.204±0.013	0.355±0.061	0.164±0.023
September	0.169±0.039	0.222±0.020	0.383±0.064	0.190±0.022
October	0.150±0.038	0.268±0.024	0.355±0.061	0.154±0.018
November	0.125±0.032	0.182±0.023	0.300±0.059	0.143±0.020
December	0.139±0.029	0.192±0.029	0.290±0.060	0.105±0.011
Summer	0.150±0.021	0.136±0.017	0.254±0.033	0.114±0.008
Autumn	0.148±0.019	0.196±0.015	0.301±0.033	0.126±0.010
Winter	0.246±0.029	0.216±0.011	0.379±0.033	0.165±0.013
Spring	0.148±0.021	0.225±0.013	0.346±0.035	0.162±0.011
Annual	0.176±0.012	0.194±0.007	0.322±0.017	0.142±0.006

Fig. 5 **Reliability diagrams for Melbourne for: (a) VRO predictions; and (b) Markov predictions for a forecast period of 12 hours. A perfect forecast is indicated by the diagonal line.**

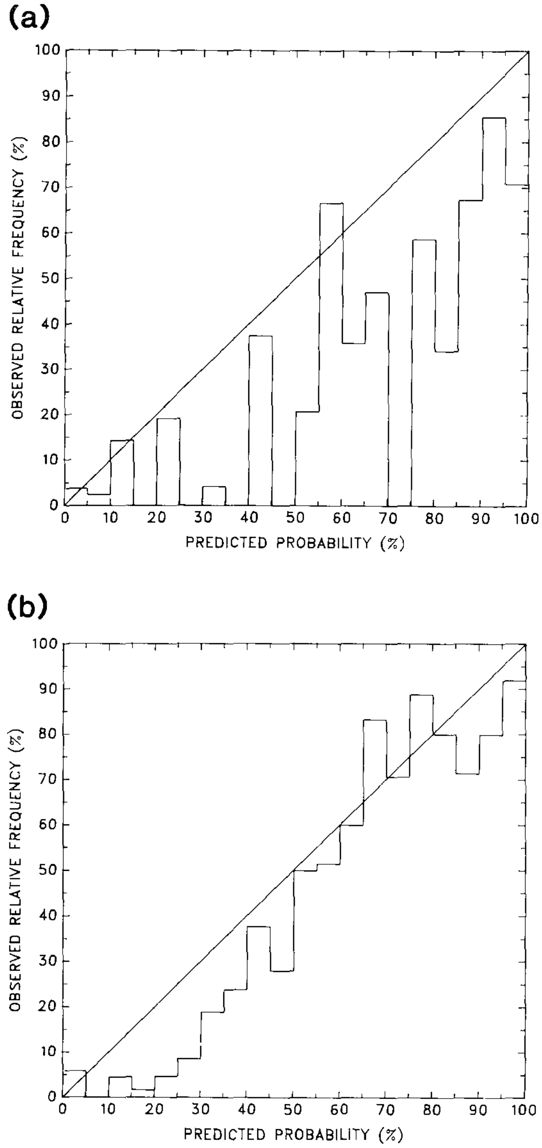


Fig. 6 **Reliability diagram for the Markov model rainfall predictions for Melbourne for a forecast period of 12 hours, where a validation period of 14 randomly selected years between 1960 and 1988 is used. A perfect forecast is indicated by the diagonal line.**

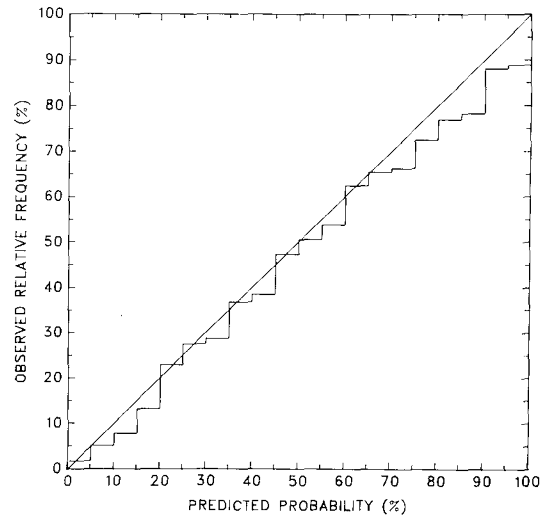
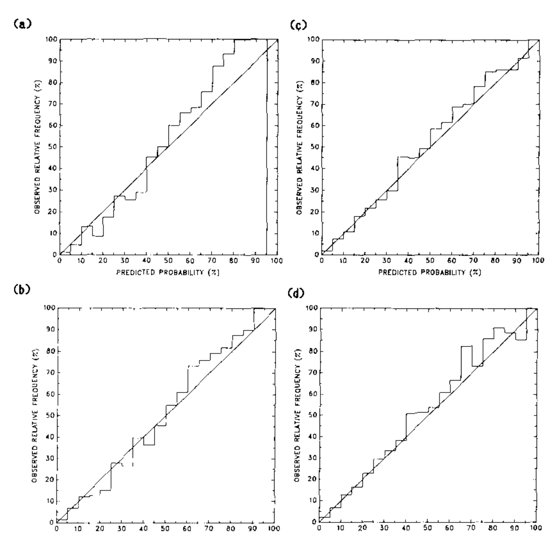


Figure 7 shows the reliability diagrams for Brisbane for forecasts of 3, 6, 9 and 12 hours. The reliability of the forecast increases with increasing length of forecast period, owing to the increase in the number of rain events. This is seen by the improvement at the high predicted probability end of the diagram. For 12 hours ahead the forecast reliability results are excellent.

Fig. 7 **Reliability diagrams for the Markov model rainfall predictions for Brisbane for: (a) 3 hours ahead; (b) 6 hours ahead; (c) 9 hours ahead; and (d) 12 hours ahead. A perfect forecast is indicated by the diagonal line.**

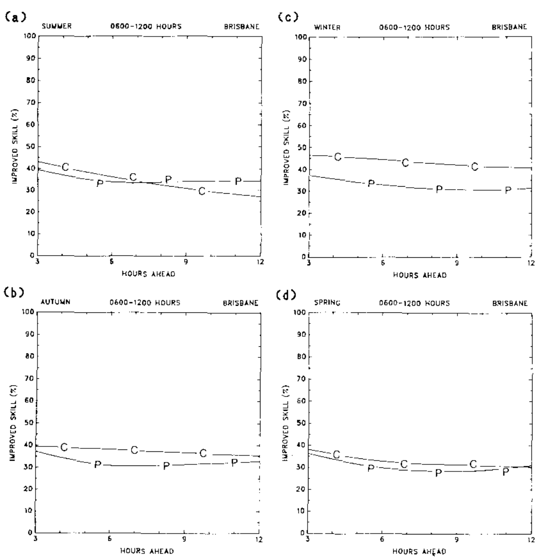


Comparisons with persistence and climatology for the capital cities

Although Markov chain models were developed for eight capital cities, because of space limitations we only present the results for three representative cities here: Brisbane, a subtropical coastal station; Canberra, a continental station and Adelaide, a southern coastal station. Results for the other cities will be published in a separate report by the Bureau of Meteorology Research Centre.

In Fig. 8 the seasonal half-Brier scores relative to climatology and persistence are shown for the validation data set; the absolute half-Brier scores are given in Table 2. There are no operational probability predictions of rain available for the 14 randomly selected years of this trial; however, the gains in the half-Brier scores relative to climatology and persistence are comparable with those obtained in the VRO trial and therefore provide convincing evidence that the method has operational utility. The diagrams are based on the average half-Brier scores computed from data with starting times of 0600, 0900 and 1200 hours for the Markov model, persistence and climatology. The label 'hours ahead' indicates the length of validity of the forecast.

Fig. 8 Percentage improvement in Markov model half-Brier scores compared to climatology (lines denoted by c) and persistence (lines denoted by p) for Brisbane for; (a) summer; (b) autumn; (c) winter; and (d) spring.



For the diagrams shown in Fig. 8 persistence is defined more stringently than above. Here persistence is defined as a prediction that the conditions (rain/no rain) which existed for the past three hours will continue to exist over the forecasting period. The improvement for the Markov model over persistence and climatology is about 30 per cent or greater, except in summer where it dips down to about 27 per cent at 12 hours ahead. We show the percentage improvement in half-Brier scores relative to those for climatology and persistence in Fig. 8, rather than the half-Brier scores themselves, because this quantity is a better measure of the performance, particularly in cases where climatology and persistence do well. As forecasts get better (that is, have smaller half-Brier scores) it becomes much harder to gain improvements. The use of relative values takes account of this difficulty.

In Fig. 9 we show the reliability diagrams for Canberra. Again the improvement at the high frequency end of the diagrams is noticeable as the length of the forecast increases. By 12 hours ahead the reliability is good.

The corresponding improvement in the seasonal half-Brier scores is given in Fig. 10. The Markov model shows an improvement compared to climatology and persistence of about 30 per cent or greater.

Figures 11 and 12 show that the model performs well for Adelaide. The results for the stations not presented were comparable to those shown.

Table 2. Seasonal half-Brier scores for 12-hour precipitation forecasts for Brisbane, Canberra and Adelaide for the validation data set.

Season	*Method*	*Brisbane*	*Canberra*	*Adelaide*
Summer	Markov	0.164	0.097	0.060
	Persistence	0.250	0.137	0.091
	Climatology	0.225	0.147	0.086
Autumn	Markov	0.125	0.086	0.102
	Persistence	0.186	0.130	0.165
	Climatology	0.193	0.130	0.161
Winter	Markov	0.083	0.114	0.153
	Persistence	0.122	0.175	0.279
	Climatology	0.141	0.163	0.230
Spring	Markov	0.109	0.114	0.106
	Persistence	0.158	0.172	0.178
	Climatology	0.156	0.176	0.163

Fig. 9 Same as Fig. 7, except for Canberra.

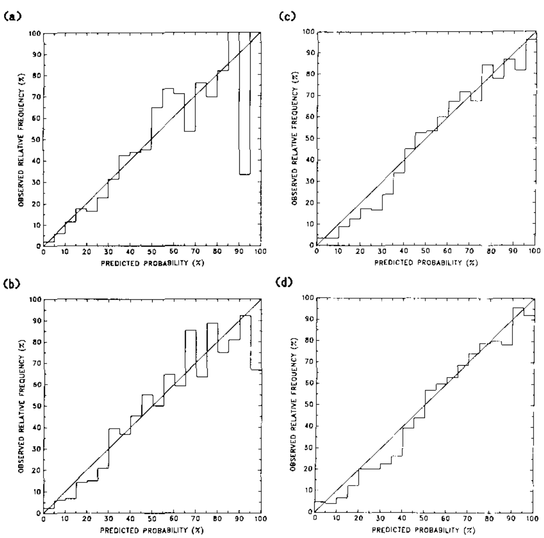


Fig. 10 Same as Fig. 8, except for Canberra.

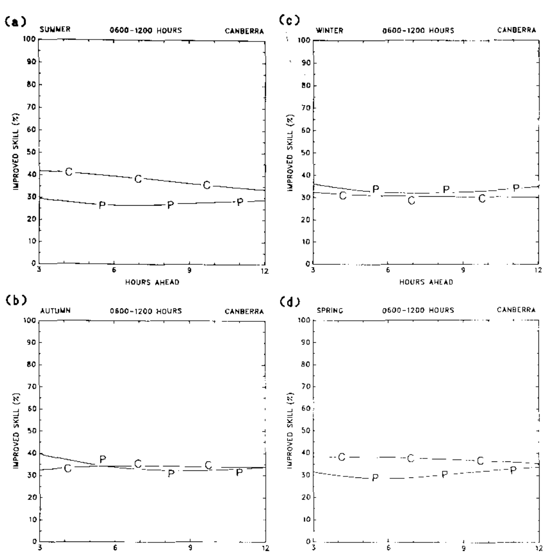


Fig. 11 Same as Fig. 7, except for Adelaide.

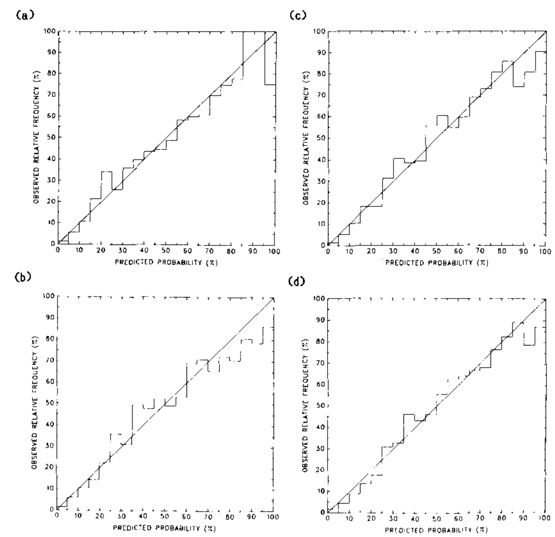
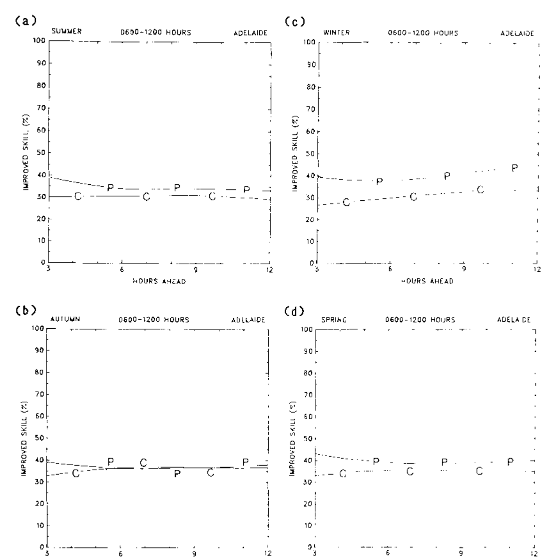


Fig. 12 Same as Fig. 8, except for Adelaide.



Discussion and conclusions

In addition to the Markov chain technique, a number of other objective techniques for short-term prediction of rainfall have been developed: nowcasting based on radar observations, MOS and NWP. Each technique has its strengths and weaknesses and time-scales for optimal performance. For example, nowcasting techniques are powerful for the 0 to 3-hour period, NWP methods and techniques based upon them, such as MOS, currently perform best beyond 12 hours. The optimal time-scale for Markov methods appears to lie neatly in the 3 to 12-hour gap.

It has been shown in the section describing the VRO trial that the Markov chain technique has sufficient skill to be used operationally either individually or as part of a larger system for the short-term prediction of precipitation at meteorological centres for which there are long records (at least 15 years) of 3-hourly surface observations.

Operations

In the first instance it is expected that the Markov chain technique will be implemented by NMC to provide 12-hour probability of precipitation forecasts for the period 6 am to 6 pm local time. These predictions will be issued to all eight RFCs. It is proposed that this will be followed by a second stage of implementation in which the Markov chain technique will be 'cycled' in 3-hourly increments around the clock so that at each observation time (e.g. 3 am, 6 am, etc.) a 12-hour forecast will be available as soon as the observations for the RFC are available on the data stream.

Further extensions of the work

In order to take advantage of the variety of objective techniques available it is becoming essential that a modern RFC develop an integrated systems approach. In this approach all the objective predictions available at the time of the forecast are blended together in an optimal fashion. For example, it has been shown by Thompson (1977), Woodcock and Southern (1983) and Fraedrich and Leslie (1987, 1988) that independent forecasting techniques, such as statistical and deterministic forecasts, or subjective and objective forecasts, can be combined in a linear manner to produce optimal error reduction. Some progress towards reaching this goal on an operational basis has already been achieved. Within the Bureau of Meteorology, systems for automated forecasting guidance have been developed (Stern 1980) and pilot schemes have been tested (Dahni et al. 1984).

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