World's Greatest Observed Point Rainfalls: Jennings (1950) Scaling Law

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ABSTRACT

The observed relation of worldwide precipitation maxima P versus duration d follows the Jennings scaling law, $P \approx d^b$, with scaling coefficient $b \approx 0.5$. This scaling is demonstrated to hold for single-station rainfall extending over three decades. A conceptual stochastic rainfall model that reveals similar scaling behavior is introduced as a first-order autoregressive process [AR(1)] to represent the lower tropospheric vertical moisture fluxes, whose upward components balance the rainfall while the downward components are truncated and defined as no rain. Estimates of 40-yr ECMWF Re-Analysis (ERA-40) vertical moisture flux autocorrelations (at grids near the rainfall stations) provide estimates for the truncated AR(1). Subjected to maximum depth-duration analysis, the scaling coefficient $b \approx 0.5$ is obtained extending for about two orders of magnitude, which is associated with a wide range of vertical moisture flux autocorrelations $0.1 < a < 0.7$.

1. Introduction

In the early 1950s, Jennings (1950) discovered a scaling law

$$P = P_0(d/d_0)^b$$

that describes the global maximum of precipitation P changing with duration d with the exponent $b \approx 0.5$, where P and P_0 are in millimeters, d and d_0 are in minutes, and $P_0/d_0^b \approx 6.75\,\mathrm{mm\,s^{-0.5}}$ (Galmarini et al. 2004). The duration corresponds to a time interval including a precipitation event that might be interrupted. His observational findings have entered hydrology textbooks and scientific papers and have since been substantiated by analyses of more station data and extended records (Eagleson 1970, Fig. 11–25; Paulhus 1965; Hubert et al. 1993; Galmarini et al. 2004; World Meteorological Organization 1986, 1994). Only recently the same analysis has been carried out for state-of-the-art climate model simulations (Zhang et al. 2013). Precipitation maxima, which are the sole subject of Jennings law, always refer to certain accumulation time scales, covering durations from minutes to 2 yr. This is a scaling law of extremes (first maxima) and not of the variability as described by variance density in a log-power–log-frequency plot or related functions in the time domain (Fraedrich and Larnder 1993). On time scales of a few days, the local thermodynamics certainly play a determining role; beyond a few days, weeks, or months, other large-scale physical factors enter. A prominent example is the Cherrapunji station in India, which is presumably related to the fact that it is in the reign of the Asian summer monsoon in a unique topographic setting and rain-bearing systems like tropical and midlatitude cyclones.

To our knowledge, only two papers have commented on this scaling: Hubert et al. (1993) connected the scaling exponent to a singularity parameter by employing multifractal methodology, while Galmarini et al. (2004)

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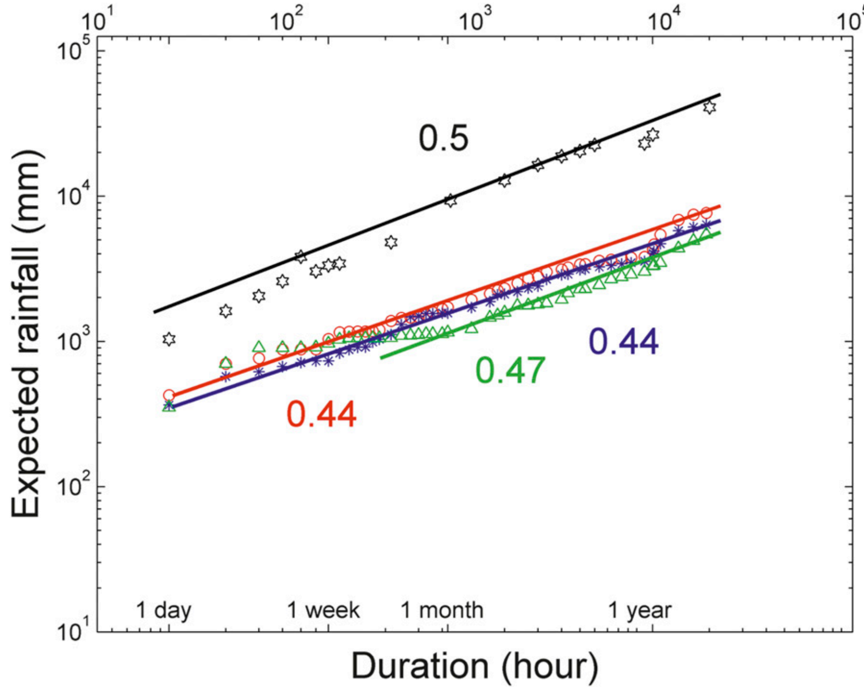

FIG. 1. Maximum precipitation-duration scaling diagram: Cherrapunji (India; pentagrams, $b \approx 0.5$), Lushan (green triangles, $b \approx 0.47$), Dongxing (red circles, $b \approx 0.44$), and Fangcheng (blue stars, $b \approx 0.44$).

proposed a combination of the precipitation distribution with a time-lag autocorrelation, thereby covering scaling ranges of about three decades of duration. As a parsimonious theoretical concept of the Jennings law, scaling of maximum rainfall depth versus duration has not yet been introduced. We like to present a simple conceptual model, the censored (or truncated) first-order autoregressive [AR(1)] process, to simulate Jennings scaling law observed in rainfall data. The censored AR(1) process is widely used as a simple and effective method to generate precipitation, as Hannachi (2013) demonstrated in a simulation producing rainfall data in Armagh, Northern Ireland. In this study, the positive values of an AR(1) process represent the vertical moisture flux in the lower troposphere. We first revisit Jennings scaling law, focusing on the scaling behavior of single-station rainfall observations (section 2). Section 3 introduces the conceptual stochastic model of rainfall and its maximum rainfall depth-duration scaling analysis. Section 4 provides conclusions and discussion.

2. Jennings scaling law

When reanalyzing the Cherrapunji (India) daily rainfall time series, Dhar and Farooqui (1973) found that the time span for maximum rainfall depth-duration scaling ranges from 1 day to 2 yr. Therefore, scaling law holds also for single stations lying in the Jennings scaling line. Along this line, it will not be surprising to find similar maximum rainfall-duration scaling at other stations, but the extent of the scaling regime may vary. This hypothesis is tested with daily precipitation time series at 732 basic weather stations over China (1951–2008, provided by the National Climate Center, China Meteorological Administration). The maximum accumulated rainfall data from 1 day to 2 yr are extracted (Fig. 1): shorter-term records ($\leq$6 days) are from Shangchuandao (1 day), Yangjiang (2 days) in Guangdong, and Lushan (from 3 to 6 days) in Jiangxi province (in central China). Longer-term (>6 days) records are mainly from Fangcheng and Dongxing in Guangxi province. As shown in Fig. 1, the maximum precipitation depth-duration relationship is observed in the selected single-station records with the scaling exponent $b \approx 0.5$. Notice that the scaling exponent b remains constant for the second and third maxima, which can be considered as the maxima found in a shorter time series. This means that the value of the scaling exponent close to 0.5 is stable with respect to the record length of precipitation data. Thus, Jennings scaling law appears as a more general scaling rule governing single-station rainfall depth-duration extremes. In the following, we introduce a conceptual model to simulate the scaling behavior in single-station data.

3. A conceptual model for Jennings scaling law

In a qualitative sense, moisture, which is supplied by surface evaporation and lateral convergence in the lower layers of the troposphere sustaining the vertical moisture flux, provides the water source for rainfall in the case of upward moisture flux (associated with the mesoscale to synoptic-scale and larger-scale airflow dynamics) and governs the dry episodes of zero rain when a downward moisture flow or zero motion is favored. Therefore, a time series of vertical moisture flow may be a suitable surrogate for a precipitation time series, if we assume, for simplicity, that only upward vertical moisture flux is proportional to rainfall rate while subsidence characterizes the zero-rainfall or dry phases. This is a basic mechanism of rain-bearing synoptic-scale systems ranging from tropical cyclones and monsoonal depressions to the midlatitude disturbances often characterized as slant-wise convection. The vertical upward motion is relevant for convection and stratiform precipitation, which are embedded in and usually forced by the developing low-pressure systems of synoptic scale with and without being effected by orography.

Here we introduce a parsimonious surrogate model for rainfall to describe the maximum depth-duration scaling following three steps.

(i) An AR(1) process is introduced as a surrogate of vertical moisture flux time series at locations near the rainfall stations analyzed in Fig. 1.

(ii) This surrogate AR(1) moisture flux time series is truncated to keep only upward fluxes representing rainfall sequences at a single station.

(iii) In the end, the truncated surrogate moisture flux time series is subjected to maximum rainfall depth-duration scaling analysis.

The moisture flux data are derived from the 40-yr European Centre for Medium-Range Weather Forecasts (ECMWF) Re-Analysis (ERA-40) datasets (1.125° grid, 1958–2001).

a. Vertical moisture flux: An AR(1) process

Based on ERA-40 datasets, daily vertical moisture fluxes m in the lower troposphere are calculated:

$$m(t) = w(t)q(t),$$

where w represents the vertical velocity and q represents the mixing ratio. ERA-40 grids are chosen to include those rainfall stations exhibiting the Jennings scaling law (Fig. 1; note Fangcheng and Dongxing are located in the same grid). The AR(1) process with discrete time steps t and Gaussian random noise r (mean μ and variance σ^2),

$$m(t) - \mu = a[m(t-1) - \mu] + r, \qquad (1)$$

is characterized by the lag-1 autocorrelation a; it corresponds to an integral time scale, $\tau \approx \sum_{i=0}^{\infty} a^i \cong 1/(1-a)$, as a suitable measure for the memory of the underlying process [see, e.g., Fraedrich and Ziehmann-Schlumbohm (1994) on surrogate predictability experiments based on AR(1)]. Figure 2 shows the lag-1 autocorrelation coefficients of water flux anomalies in the lower troposphere at the selected grids and their AR(1) processes. Notice that for Fangcheng and Lushan, most maximum rainfall events happen during the rainy season. Therefore, we use the water vapor flux data in the rainy season to reduce seasonal fluctuations. All of them show short-term memory (less than 4 days), which leads to the respective AR(1) processes. The next step is to treat an autoregressive process as a surrogate of the sequences of positive and negative moisture fluxes (or updraft versus subsidence).

b. Rainfall: A truncated AR(1)

Rainfall rate $R(t)$ can be estimated to be proportional to the vertical flux of moisture:

$$R = Ewq, \quad w > 0,$$

where $w > 0$ is the ascent rate and E is the precipitation efficiency, which is defined as the ratio of the mass of water falling as precipitation to the influx of water vapor mass into the cloud and supposed constant (see, e.g., Doswell et al. 1996). Then the total precipitation is formalized as $p = Rt_d$, with the precipitation duration t_d. On the basis of this premise, we assume a proportionality between the amplitude of the daily moisture updraft $m(t)$ and the expected value of daily rainfall $p(t)$. The surrogate precipitation time series—suitable for statistical analysis—is simply given by the positive values of the moisture flux $m(t) > 0$, while the negative values $m(t) < 0$ represent zero rainfall:

$$p(t) = \begin{cases} Em(t) & \text{for} \quad w > 0 \\ 0 & \phantom{\text{for}} \quad w \le 0 \end{cases}, \qquad (2)$$

where E is a constant for precipitation efficiency setting $E = 1$ in our analysis (units of both p and m are millimeters per day). This model generates a truncated stochastic time series that is based on a continuous autoregressive process to model intermittent phenomena (see Hannachi 2013 for a comprehensive analysis, application, and review). The choice of short-term memory and autoregressive-type stochastic models for rainfall

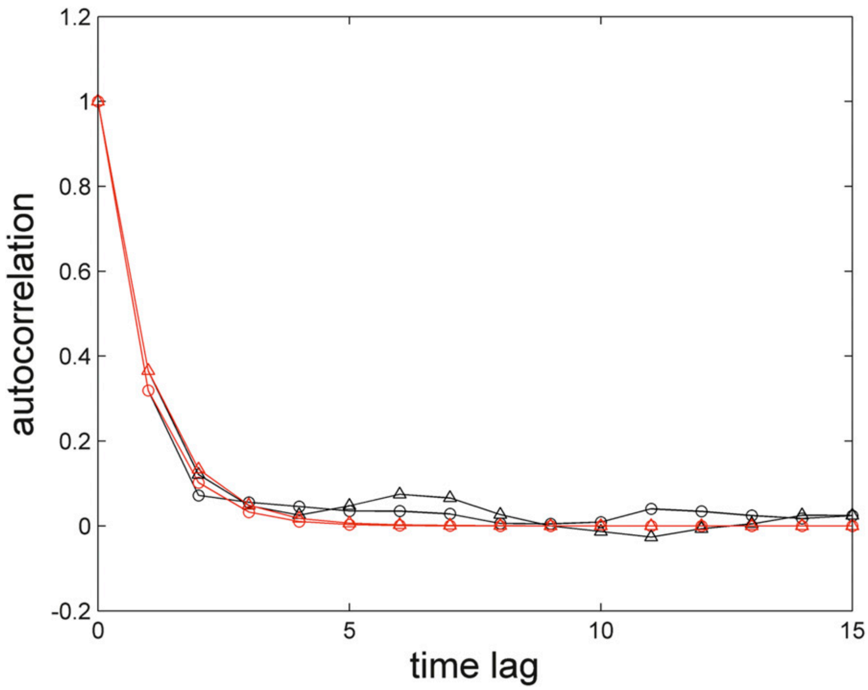

FIG. 2. Autocorrelation functions (full lines) of 850-hPa vertical water flux anomalies at grids close to the following stations (black) and their AR(1) processes (red): Lushan (triangles, $a \approx 0.36$) and Dongxing and Fangcheng (circles, $a \approx 0.31$).

surrogates has been substantiated further by observed scaling properties of daily precipitation records (see, e.g., Fraedrich and Larnder 1993; Fraedrich et al. 2009).

c. *Maximum rainfall depth-duration scaling for a truncated AR(1)*

An example of an AR(1) process is shown in Fig. 3 (left) for the station Fangcheng–Dongxing ($a \approx 0.31$), where the positive part represents upward water vapor flux or rainfall intensity. Figure 3 (right) shows the associated maximum rainfall depth-duration scaling (or Jennings scaling law) suggesting a power law exponent close to $b \approx 0.5$, which lasts from days to 3 months. Because of the limited length of the rain season here, the duration cannot be longer than 4 months. The mean of expected rainfall in Fangcheng is 51.1 mm day^{-1}, compared to 11.9 mm day^{-1} in the observations; the standard deviation of expected rainfall is 55 mm day^{-1}, compared to 30.3 mm day^{-1} in the observations. The same analysis has also been done for station Lushan ($a \approx 0.36$; Fig. 4). The mean of expected rainfall in Lushan is 32 mm day^{-1}, compared to 7.1 mm day^{-1} for the observed in the rain season, and the standard deviation of expected rainfall is 39 mm day^{-1}, compared to 21 mm day^{-1} for the observed in the rain season. The average and variance of the rainfall in Fangcheng is higher than in Lushan. Since the precipitation efficiency cannot be 100%, the magnitudes of simulated rainfall are much higher than those of the observed ones. AR(1) processes are capable of reproducing the Jennings scaling law in single stations.

Supposing the moisture flux is a zero-mean, unit variance AR(1) process ($\mu = 0, \sigma = 1$), Eq. (1) becomes $m(t) = a \times m(t-1) + r$, the maximum rainfall depth-duration relationship is extracted from the positive part of this truncated AR(1) process. As shown in Fig. 5, the Jennings scaling law with power law exponent close to $b \sim 0.5$ covers about two orders of magnitude. Note that the power law scaling does not change substantially for different coefficients $0.1 < a < 0.7$. However, for larger integral time scales (for example $a = 0.999$) the power law slope increases to $b \approx 0.8$. The results stay robust for second and third maxima. In this sense, we may interpret the Jennings law scaling as an outcome of an AR(1) process. The calculations above are based on a constant efficiency, $E = 1$. Calculations for varying E and with shorter memory show that the scaling exponent does not change (figures not shown).

4. Conclusions and discussion

The Jennings scaling law, $P \approx d^b$ with $b \approx 0.5$, has been revealed from a worldwide ensemble of rainfall

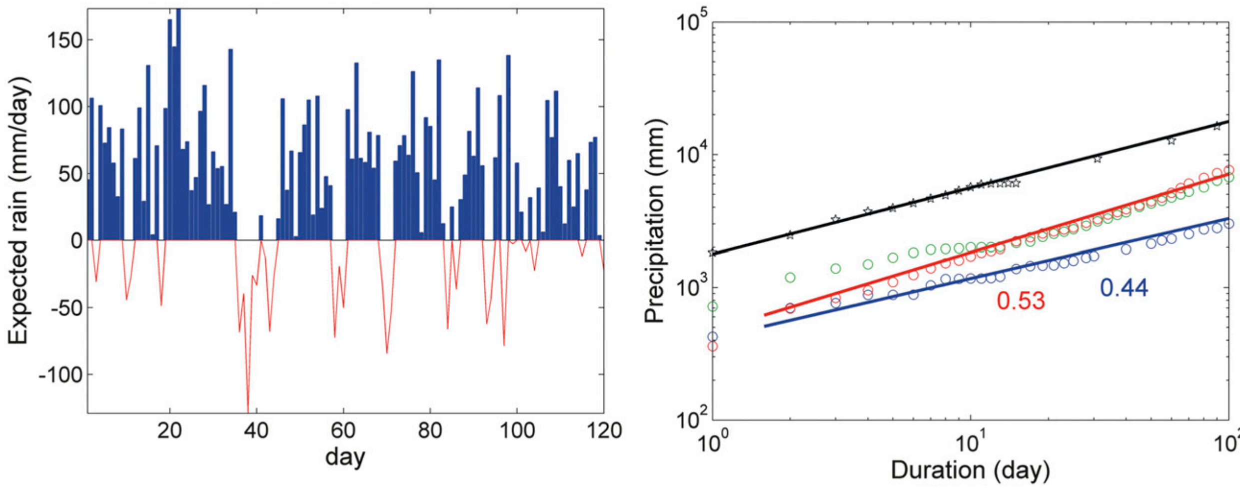

FIG. 3. (left) Snapshot of an AR(1) process time series at station Fangcheng ($a \approx 0.31$) with positive ranges marked in blue. (right) Simulated maxima rainfall (red cycles) vs duration relationship by the positive parts of AR(1) processes at Fangcheng station. Blue circles are observed maxima, green circles are expected rainfall, and black pentagrams are world records.

observations (Jennings 1950). The finding has been substantiated for three decades of the scaling regime when the analysis is confined to daily rainfall records (taken from China's basic weather stations). As a concept for such station-related maximum rainfall depth-duration scaling behavior, a truncated (censored) first-order autoregressive process is introduced, which has recently also been used to simulate daily precipitation times series in midlatitudes (Hannachi 2013). Here we provide the physical censor to truncate the downward episodes of an AR(1) process for the case of downward lower troposphere moisture fluxes. The remaining upward-only moisture flux time series, sustaining the rainfall events, describes the rainfall intermittency and shows the scaling behavior of the maximum rainfall depth-duration following Jennings scaling law as observed at single rainfall stations.

In this sense, we have introduced the dynamics behind the underlying AR(1) process as a surrogate model for atmospheric water fluxes; by implementing the censorship truncating the downward fluxes, only the positive (upward) fluxes are kept and are thus intermittent, which leads to the nonlinear scaling behavior documented by the Jennings scaling law.

The scaling exponents of the first, second, and third maxima in ERA-40 are found close to $b = 0.7$ (figure not shown). As we discussed in Zhang et al. (2103), this scaling exponent is similar to the one in the ECHAM5–Max Planck Institute Ocean Model simulations with lower resolution (T31), which means that the data are

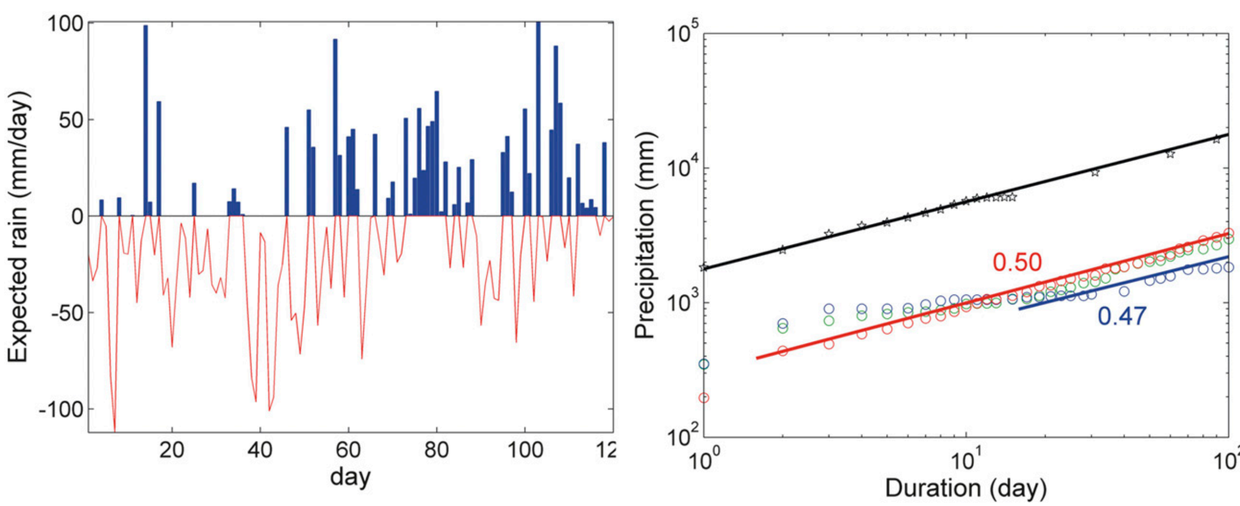

FIG. 4. As in Fig. 3, but for the station Lushan ($a \sim 0.36$).

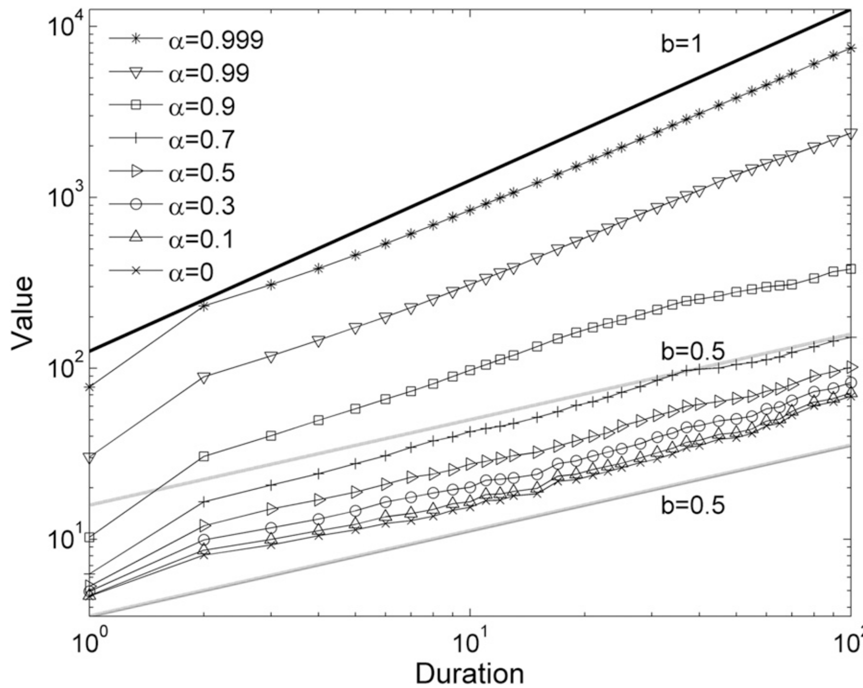

FIG. 5. Simulated maximum rainfall vs duration relationships obtained by the positive parts of AR(1) processes with different coefficients *a* as indicated (total length is 10^6 time steps). The dotted and solid lines denote the exponent $b = 0.5$ and $b = 1$, respectively.

long-range autocorrelated. The assimilation model used for the ERA-40 data has a horizontal spectral resolution of T159 and L60 height levels. The reason why ERA-40 rainfall data behave similar to low-resolution (T31) rainfall simulations is still unknown.

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REFERENCES

Dhar, O. N., and S. M. T. Farooqui, 1973: A study of rainfalls recorded at the Cherrapunji Observatory. *Hydrol. Sci. Bull.,* **18,** 441–450, doi:10.1080/02626667309494059.

Doswell, C. A., III, H. E. Brooks, and R. A. Maddox, 1996: Flash flood forecasting: An ingredients-based methodology. *Wea. Forecasting,* **11,** 560–581, doi:10.1175/1520-0434(1996)011<0560:FFFAIB>2.0.CO;2.

Eagleson, P. S., 1970: *Dynamic Hydrology*. McGraw-Hill, 200 pp.

Fraedrich, K., and C. Larnder, 1993: Scaling regimes of composite rainfall time series. *Tellus,* **45A,** 289–298.

——, and C. Ziehmann-Schlumbohm, 1994: Predictability experiments with persistence forecasts in a red-noise atmosphere. *Quart. J. Roy. Meteor. Soc.,* **120,** 387–428, doi:10.1002/qj.49712051608.

——, R. Blender, and X. Zhu, 2009: Continuum climate variability: Long-term memory, scaling, and 1/f-noise. *Int. J. Mod. Phys. B,* **23,** 5403–5416, doi:10.1142/S0217979209063729.

Galmarini, S., D. G. Steyn, and B. Ainslie, 2004: The scaling law relating world point-precipitation records to duration. *Int. J. Climatol.,* **24,** 533–546, doi:10.1002/joc.1022.

Hannachi, A., 2013: Intermittency, autoregression and censoring: A first-order AR model for daily precipitation. *Meteor. Appl.,* doi:10.1002/met.1353, in press.

Hubert, P., Y. Tessier, S. Lovejoy, D. Schertzer, F. Schmitt, P. Ladoy, J. P. Carbonnel, and S. Violette, 1993: Multifractals and extreme rainfall events. *Geophys. Res. Lett.,* **20,** 931–934, doi:10.1029/93GL01245.

Jennings, A. H., 1950: World’s greatest observed point rainfalls. *Mon. Wea. Rev.,* **78,** 4–5, doi:10.1175/1520-0493(1950)078<0004:WGOPR>2.0.CO;2.

Paulhus, J. L. H., 1965: Indian Ocean and Taiwan rainfalls set new records. *Mon. Wea. Rev.,* **93,** 331–335, doi:10.1175/1520-0493(1965)093<0331:IOATRS>2.3.CO;2.

World Meteorological Organization, 1986: Manual for estimation of probable maximum precipitation. 2nd ed. Operational Hydrology Rep. 1, WMO 332, 269 pp. [Available online at http://library.wmo.int/pmb_ged/wmo_332.pdf.]

——, 1994: Guide to hydrological practices. 5th ed. WMO 168, 402 pp. [Available online at ftp://ftp.wmo.int/Documents/MediaPublic/Publications/Guide_to_Hydrological_Practices/WMOENG.pdf.]

Zhang, H., K. Fraedrich, R. Blender and X. Zhu, 2013: Precipitation extremes in CMIP5 simulations on different time scales. *J. Hydrometeor.,* **14**, 923–928, doi:10.1175/JHM-D-12-0181.1.