Stochastics and Dynamics

FICKIAN DIFFUSION AND NEWTONIAN COOLING: A CONCEPT FOR NOISE INDUCED CLIMATE VARIABILITY WITH LONG-TERM MEMORY?

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Observed near surface air and soil temperature time series reveal the same long-term memory, which is associated with a power-law scaling of the frequency spectra, $S(\omega) \sim \omega^{-\beta}$ with $\beta \sim 0.6$, lying between white and flicker noise, $0 < \beta < 1$. As this power law scaling is globally observed and not consistent with the Brownian motion concept of climate variability, Fickian diffusion is added to a Newtonian cooling relaxation to provide a more suitable analog of climatic fluctuations: (i) Diffusive plus random heat fluxes parameterise the turbulent mixing by synoptic scale eddy life cycles, affect tropospheric and near surface soil temperatures and excite a long-term memory regime with a $\beta \sim 0.5$ scaling. (ii) Newtonian cooling describes the near surface soil temperatures relaxing towards a global mean deep soil temperature and stabilises the system to a white noise response at very low frequencies. The long-term memory regime emerges from the high frequency scaling ($\beta \sim 1.5$), once temperatures become correlated in space due to diffusion, so that spatially averaged fluctuations correlate for times beyond the diffusion time scale. The long-term memory regime disappears into a white noise plateau ($\beta \sim 0$), when low frequencies exceed the damping time scale of Newtonian cooling. This system may be interpreted as a diffusive system relaxing towards the deep soil restoration temperature with an almost infinitely large time scale.

1. Introduction

In the mid-70ies the Brownian motion entered climate research as an analog for the Earth's climate fluctuations (Hasselmann 1976), which led to an intensive red noise search in data and comprehensive general circulation models (GCM) alike. At the same time observations and modeling of flicker noise and other power-law scaling regimes emerged (for example, Voss and Clark 1976, van Vliet et al. 1980) with concepts as close to the climate system's energy balance as the Brownian motion analog: The stochastic fluctuations, which are added to a linear system relaxing to a stable equilibrium, are replaced by stochastic fluxes, whose divergence is added to a linear system with Fickian diffusion. That is, the radiative-convective

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processes, parameterised by a relaxation to the greenhouse equilibrium state plus additve noise, are replaced by the divergence of a heat flux, parameterised by a diffusive plus a noisy flux.

Since then power-law power spectra different from Brownian motion have been identified in observed records and model simulations of the climate system. For example, the near surface temperature (Manabe and Stouffer 1996, Pelletier 1997) shows low-frequency behaviour which does not, up to very long periods, asymptote towards a white plateau. While most of these studies (for a review see Pelletier and Turcotte 1999) are guided by self-affine scaling laws governing the dynamics of a non-linear system, the associated long-range memory or correlation aspect has been emphasised only recently (Koscielny-Bunde et al. 1998, Talkner and Weber 2000 analysing observed temperatures, Müller et al. 2001 analysing simple GCM simulations). In these studies, the variance spectrum analysis has been replaced by methods applied to a random walk profile (that is, time series of accumulated anomalies) to determine the scaling laws and long-term memory, that is fluctuation and detrended fluctuation analysis (FA and DFA, Peng et al 1994). The analyses suggest that the near surface temperature fluctuations are governed by a universal scaling behaviour showing long-term memory correlations up to at least thirty years. This feature has been attributed to the slow ocean and cryosphere dynamics also affecting the atmosphere as the fastest of the climate components. A quantitatively similar long-term memory of global angular momentum has been found in a GCM driven by Newtonian cooling and damped by Rayleigh friction (Müller et al. 2001), which may be due to other mechanisms.

Both the observational support and an understanding of stationary climate processes with long-term memory is far from being complete, not to mention the existence of a generally accepted conceptual model. This note presents additional information on the low-frequency behaviour of two variables of the heat balance: The near surface air and soil temperatures and precipitation are analysed using observations and simulations. First, the analysis method is briefly described (Section 2) and then applied to long time series of the near surface air and soil temperature (Section 3). Finally, a simple conceptual model comprising Fickian diffusion and Newtonian cooling is analysed, which reveals a long-term memory regime (Section 4).

2. Scaling regimes: Correlations, fluctuation analysis and power spectrum

A time series y(j) is interpreted by the random walk positions or the Z(j)-profile running through time j, where $Z(j) = z_1 + z_2 + ... + z_j$. A random step (or anomaly), $z_j = y_j - <y>$, is obtained after subtracting the ensemble mean annual cycle $<y>$. The fluctuation analysis applied to the random walk profile is based on the structure function approach (Monin and Yaglom 1975) to obtain scaling properties

$$F(r) = [(F_1^2 + F_2^2 + ... + F_N^2)/N]^{1/2}$$

Steps or squared distances, $F_m^2(r) = (Z_{(m)r+1} - Z_{(m-1)r+1})^2$, change with segment-length r and are averaged over all segments of the profile, that is from m = 1 to N.

Stationarity and normalisation by the total variance σ^2 yields the following relation between auto-correlation and structure function, $C(r) = 1 - \frac{1}{2}F(r)$. Thus the power-law scaling of the auto-correlation corresponds to a power-law scaling of the structure function

$$C(r) \sim r^{-\gamma} \text{ and } F(r) \sim r^{\alpha}$$

with $\alpha = 1 - \frac{1}{2}\gamma$. Fourier transformation of the correlation function leads to the variance spectrum, which is also governed by a power-law, $S(\omega) \sim \omega^{-\beta}$ with $\beta = 1-\gamma = 2\alpha-1$. Likewise fractal dimension, $d = \alpha-\frac{1}{2}$, and Hurst exponent, $H = \alpha-1$, can be deduced. Note that stationary processes without diverging moments but with long-term memory are limited in their range of power-law exponents: $0 < \gamma < 1$ or $1 > \alpha > \frac{1}{2}$ or $1 > \beta > 0$. That is, short- or long-term memory can simply be distinguished by the integral timescale $T = \int_0^\infty C(r)dr$ being finite or infinitely large. The range is limited by white noise ($\gamma = 1$, $\alpha = \frac{1}{2}$, and $\beta = 0$), which is stationary without memory (zero or finite integral time-scale) and by Flicker- or 1/f-noise ($\gamma = 0$, $\alpha = 1$, and $\beta = 1$), which is non-stationary with long-term memory (infinite integral time scale). For example, a pure random walk attains $\gamma = -1$, $\alpha = \frac{3}{2}$, and $\beta = 2$.

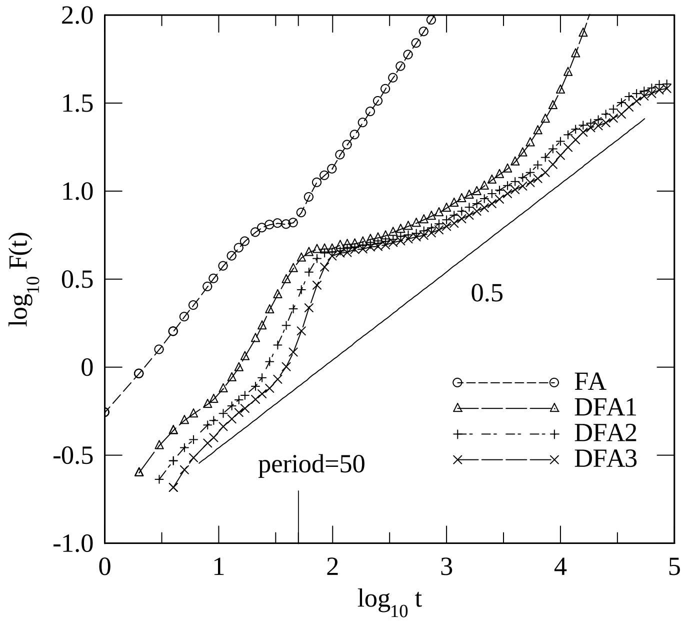

Fig. 1. Fluctuation and detrended fluctuation analyses (DFA-1 to 3) of an artificial time series: Superimposition of a linear trend, a 50day wave, and white noise.

Detrended fluctuation analysis (DFA): Replacing squared distances $F_m^2(r) = (Z_{(m)r+1} - Z_{(m-1)r+1})^2$ of all time-segments by the variances $F_m^2(r) = var(Z'_{(m)r+j})$ within the segments does not affect the power-law scaling. Detrended fluctuation analysis removes the linear, quadratic, cubic etc. trends from the segments (see

Peng et al. 1994, Bunde et al. 1999, Heneghan and McDarby 2000) by fitting linear or higher order polynomials to all segments of the profile before the segmential variances are determined. A surrogate data set is analysed, which serves as an introduction to the methodology. Figure 1 shows the fluctuation analysis by the structure function (FA) and the detrended fluctuation analysis employing linear, quadratic and cubic polynomials (DFA-1, DFA-2, and DFA-3) applied to an artificial time series, which consists of a linear trend plus a periodic 50day-wave plus a first order auto-regressive or red noise process. The periodic signal is captured by all methods. Only the detrended fluctuation analysis, employing trend-removing polynomials of second and higher order, identifies the long-term fluctuation feature of the time series. As FA and DFA-1 do not scale with the white noise power law, $\alpha = \frac{1}{2}$, DFA-2 will be used for further analysis.

3. Application: Near surface air and deep soil temperatures

Temperature time series are subjected to the DFA analysis to identify regimes of long-term memory. The data are chosen for the station Potsdam (Germany, 1893-2001) with high resolution near surface air and soil temperature records being available for a long time period (Figure 2). The near surface air temperature time series (observed and GCM-simulated) in other climate regions show similar behaviour.

Near surface air and soil temperature: The observed near surface air temperature (2m above ground) follows a power-law scaling regime over two decades from less than one month to about 10,000 days (or 30 years). The DFA-scaling with $\alpha \sim 0.7$ corresponds to a power-law variance spectrum $\beta \sim 0.4$ (plotted in Figure 2 with increasing log of the period). It indicates a long-term memory induced by the persistence of temperature anomalies. This regime has been documented at many other weather stations, but appears to be break down at long periods. Whether it indicates a transition to a new scaling regime ($\alpha = 0.9$ or $\beta = 0.8$) regime, to a very long quasi-periodic oscillation, or whether it occurs simply because the results are less reliable for very long time scales near the length of the time series, cannot be resolved by the data. This long-term memory regime, $\beta \sim 0.4$, is also found in the tropospheric temperatures simulated by a thousand year run of a state of the art coupled atmosphere-ocean GCM (ECHAM/HOPE). And it is discovered in the subsurface layer observation down to about 2m soil deepth. Moreover, this scaling breaks down in deeper layers and changes towards a transitional flicker noise regime. The almost global universality of the long term memory regime, its beginning at time scales of a month or less, and its weak relaxation towards a slow climate compartment suggest the cooperation of some fundamental processes. It should be noted that a globally representative standard deviation, $\delta\alpha \sim \pm 0.1$, which corresponds to $\delta\beta \sim \pm 0.2$, has been estimated from ten century samples of all grid-point DFAs, which demonstrates a not so constant scaling behaviour, at least in global climate models and station data. Some qualitative concepts are introduced.

Fickian diffusion (troposphere): The global extent of the long term memory regime suggests that the meridionally diffusive flux, $D\theta_y$, is the underlying mechanism. It provides the equator and pole heat exchange, to which synoptic scale eddies contribute the largest amount. This is a standard parameterisation of atmospheric

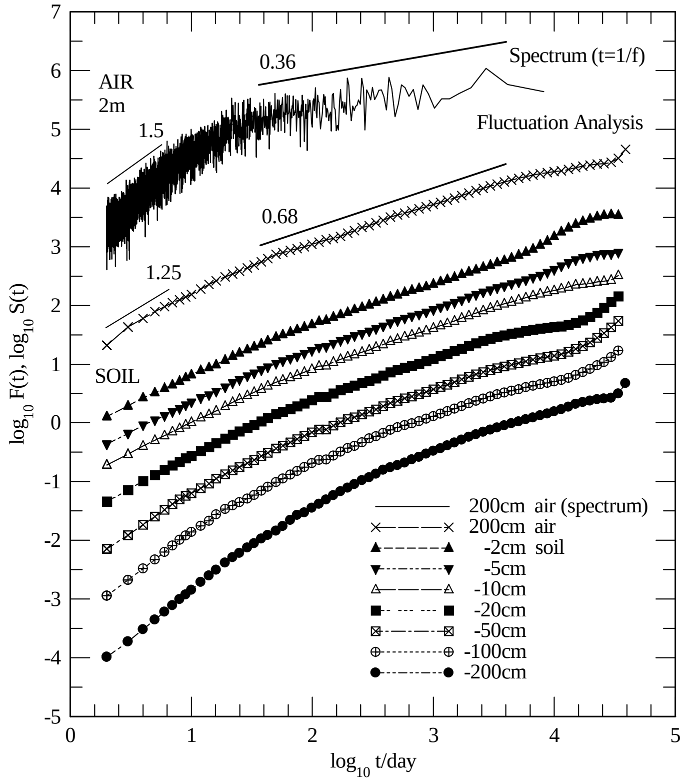

Fig. 2. Single station near surface temperature time series: Detrended fluctuation analysis (DFA-2) and variance spectrum of the near surface air and soil temperatures at Potsdam (Germany, daily data from 1893 to 2001). Stations in other climates show similar results.

heat transport and used in many energy balance climate models. The diffusivity of synoptic scale eddies is defined by the time and the space affected by their life cycle. Noise is added to the meridional heat flux; it may be induced by the water cycle, because precipitation is observed to be predominantly white from months to decades (see Fraedrich and Larnder 1993, Olsson et al. 1999). The tropospheric mixing by eddy or blocking life cycles generates almost homogeneous temperature domains, which extend over a large meridional scale in the troposhere and downward into the underlying near surface soil. This has the following two consequences: (i) Mixing affecting the troposphere and the near surface soil are treated as a single compartment being in contact through the boundary layer processes. This is parameterised by integrating the joint temperature over a domain affected by the synoptic scale life cycle which, in the meridional direction, is the Rossby radius of about 1000km. (ii) Newtonian relaxation influences the near surface soil (now linked with the troposphere) which, with its almost infinite heat capacity, relaxes very slowly to the deep soil with its hardly changing restoration temperature T_E. Note that a mixed layer or dynamical ocean has the same effect so that, in the following, the soil may also be replaced by a mixed layer ocean.

4. Eddy diffusive plus random fluxes in a climate system damped by Newtonian cooling

A reaction-diffusion equation is used as a conceptual model to describe the meridionally and vertically integrated energy balance climate in terms of the tropospheric or near surface soil temperature, $T = \theta + T_E$, fluctuating about a globally averaged deep soil/ocean restoration temperature T_E. These temperature fluctuations are induced by two processes: (i) Meridional eddy heat fluxes in the troposphere are parameterised by Fickian diffusion plus a random volume noise, $F = -D\theta_y - F'$. (ii) The near surface soil relaxing towards the equilibrium deep soil temperature is described by Newtonian cooling, $-\frac{\theta}{\tau}$.

$$\theta_t = -\frac{\theta}{\tau} + (D\theta_y + F')_y$$

The random heat flux and its intensity, $<F'(y,t)F'(y',t')>= 2\pi F_0^2 \sigma(y-y')\sigma(t-t')$ with $dim(F_0^2) = K^2 m^3 s^{-1}$, may be due to the randomly fluctuating water-cycle. The Fickian diffusion represents the eddy heat fluxes as the most prominent mixer of lower tropospheric temperature contrasts, where the diffusion coefficient $D = <V\Delta y>$ is scaled by meridional particle displacement, $\Delta_y = 2L$, and the life-time of synoptic eddies, $t \sim 10 days$, by the mixing time scale. This defines the short-term limit of the climate system's memory, which is of the magnitude of the life cycle of the North-Atlantic Oscillation (NAO). With the Rossby-radius as mixing-length, L=1000km, and the advective time-scale, V = L/t, one obtains $D \sim 2L^2/t \sim 2 \cdot 10^6 m^2 s^{-1}$ (see also Green 1970). The analysis follows, in parts, Voss and Clarke (1976).

Fourier transformation of the temperature anomaly $\theta(k,\omega)$ in (k,ω)-space and the response spectrum $S(k,\omega) = <\theta\theta^*>$ with the conjugate complex θ^* yields

$$\theta(k,\omega) = \frac{ikF_0(k,\omega)}{(Dk^2 + \tau^{-1} - i\omega)}$$

$$S(k,\omega) = \frac{F_0^2 k^2}{[(Dk^2 + \tau^{-1})^2 + \omega^2]}$$

The mean mixing temperature $[\theta]$ is obtained after integration over the 2L mixing-domain. It is controlled by the diffusion due to eddy life-cycles and describes the tropospheric and near surface soil temperature:

$$[\theta(t)] = (2L)^{-1} \int_{-L}^{L} dy\theta(y,t)$$

$$[\theta(\omega)] = (2L)^{-1} \int_{-L}^{L} \theta(y,\omega)dy = (2\pi)^{-1} \int_{-\infty}^{\infty} sin(kL)\theta(k,\omega)(kL)^{-1}dk$$

The variance spectrum $S(\omega)$ of the temporal fluctuations of the temperature averaged over the mixing domain gives

$$
\begin{aligned}
S(\omega) &= \frac{F_0^2}{4\pi^2}\int_{-\infty}^{\infty} dkk^2 sin^2(kL)[(Dk^2+\pi^{-1})^2+\omega^2]^{-1}(kL)^{-2} \\
&= \frac{F_0^2}{4\pi^2}L^{-2}\cdot\pi D^{-1/2}(\tau^{-2}+\omega^2)^{-3/4}[sin(\frac{1}{2}\phi) \\
&\quad -exp(-\beta)[cos(\frac{1}{2}\phi)sin(\alpha)+sin(\frac{1}{2}\phi)cos(\alpha)]]/sin(\phi)
\end{aligned}
$$

where $dim(S) = K^2s$ and $\phi = arctan(\tau\omega)$, $\alpha = 2L\mu sin(\frac{1}{2}\phi)$, $\beta = 2LD^{-1/2}(\omega^2 + \tau^{-2})^{-1/4}cos(\frac{1}{2}\phi)$. The spectrum reveals two fundamental time scales (or frequencies), which govern the deterministic dynamics of the system's mixing-length averaged temperature fluctuations: (i) Newtonian cooling characterises the slow relaxation process towards the deep soil/ocean restoration temperature and, therefore, the temperature relaxation time scale. (ii) The diffusive mixing, $\omega_0 = \frac{1}{2}D/L^2 \sim \frac{1}{10days}$, is short due to the fast mixingprocesses in the lower atmosphere. These time scales lead to three spectral regimes (Figure 3): (a) High frequencies show a slope $(\omega_0 \ll \omega) : S(\omega) \sim \omega^{-3/2}$. The contrast to the red noise regime or -2 power-law may be attributed to the noise introduced as a divergence and/or to the spatial averaging. (b) Intermediate frequencies are characterised by long-term memory effects, which are limited by the time scale of the slow climate component $(\omega_\tau \ll \omega \ll \omega_0) : S(\omega) \sim \omega^{-1/2}$. (c) At very low frequency, the white noise plateau is reached $(\omega_\tau \ll \omega \ll \omega_0) : S(\omega) \sim \omega^0$. The white noise response level at zero frequency is

$$
S(\omega=0) = \frac{F_0^2}{4\pi^2}L^{-2}\cdot\frac{1}{4}\pi D^{-1/2}\tau^{3/2}[1-exp(-\beta)]
$$

Special case $\tau \to \infty$: Neglecting Newtonian cooling ($\tau \to \infty$), that is $\mu \to \omega^{1/2}, \phi \to \frac{1}{2\pi}$, $\alpha = \beta \to a(2\omega)^{1/2}$, means that the system cannot be attracted by the stable equilibrium but the variance density increases $S(\omega) \to \infty$ for $\omega \to 0$:

$$
\begin{aligned}
S(\omega) &= \frac{F_0^2}{4\pi^2}\int_{-\infty}^{\infty}\frac{k^2 sin^2(kL)}{(D^2k^4+\omega^2)(kL)^2}dk \\
&= (\frac{F_0^2}{4\pi^2})L^{-2}\cdot\pi D^{-1/2}(2\omega)^{-3/2}[1-exp(-Z)[sin(Z)+cos(Z)]]Z^{-3}
\end{aligned}
$$

The frequency is normalised by the natural one associated with the diffusion process, ω_0 with $Z^2 = \omega/\omega_0$. This special case reveals two power-law scaling regimes: For low frequencies $(\omega < \omega_0)$ the spectrum follows $S(\omega) \sim \omega^{-1/2}$ and for higher frequencies $(\omega > \omega_0)$ one obtains $S(\omega) \sim \omega^{-3/2}$. This system is similar to the one analysed by Voss and Clarke (1979) and has been adopted as a paradigm for the observed power-law or self-affine scaling of the climate system's low-frequency fluctuations by Pelletier and Turcotte (1999). However, they use the vertical heat flux divergence associated with vertical overturning and a vertical mixing length, which affects the whole depth of the atmosphere.

Resonance: It appears that the long-term memory of the near surface temperatures influences the diffusive energy flux affecting the length 2L of a mixing domain.

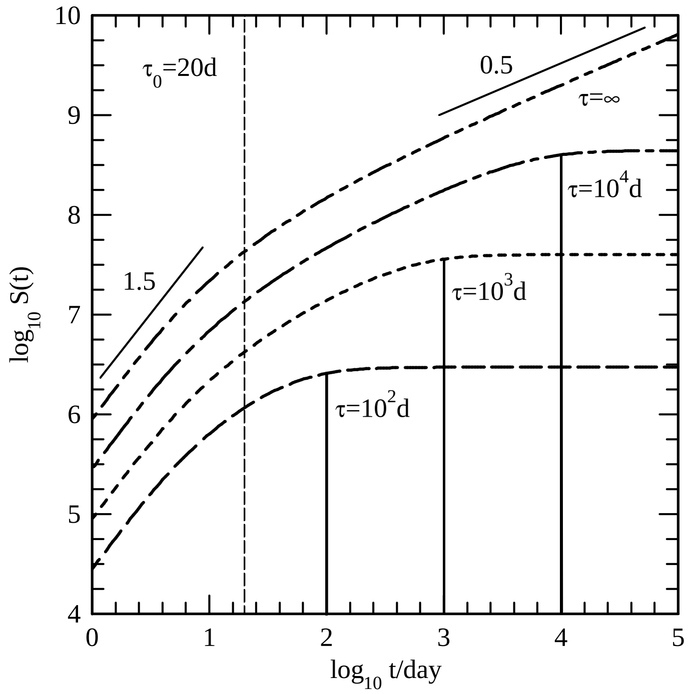

Fig. 3. Normalised variance spectra of the mixing-length averaged temperature anomaly fluctuations in a simple energy balance climate model, in which Newtonian cooling stabilises the divergence of one-dimensional Fickian diffusive plus white noise fluxes: The diffusivity and mixing length is fixed at $D = 10^6 m^2/s$ and L = 1000km, the Newtonian relaxation times (in days) vary, $\tau = 10^2, 10^3, 10^4$, and ∞.

That is, the temperature fluctuations (averaged over the 2L-domain) are long-term correlated for low frequencies, $\omega < \omega_0 = \frac{1}{2}D/L^2 \sim \frac{1}{10days}$, smaller than the natural frequency representing the diffusion process. This may be interpreted as resonance (Benzi et al. 1982 and 1989), affecting the system at periods, which are long enough for diffusion or mixing to spatially correlate temperatures across the mixing length 2L. The domain averaged temperature fluctuations become correlated. This is similar to a positive feedback, so that a long-term memory regime evolves with a process related power-law spectrum, which extends over a wide frequency band. The low frequency limit of this regime occurs at the relaxation time-scale, ω_τ, due to the stabilising influence of Newtonian cooling. The diffusion time scale defines the high frequency limit, ω_0, up to which temperature fluctuations at the boundaries of the mixing domain remain independent of one another and a long-term memory of the spatially averaged temperature fluctuations cannot evolve.

5. Conclusions

Observed near surface air and soil temperature time series reveal a long-term memory regime of the climate system characterised by a self-affine or power-law scaling. In the frequency domain the spectrum, $S(\omega) \sim \omega^{-\beta}$ with $\beta \sim 0.6$, lies between white and flicker noise, $0 < \beta < 1$. Similar scaling behaviour, determined by variance spectra and detrended fluctuation analyses, occurs in many climate regions. This scaling is not consistent with Brownian motion, which has been conveniently adopted as a concept to explain red noise climate variability. Fickian diffusion appears to be a more suitable analog to describe the heat balance of the climate system, which leads to long-term memory: The diffusive and random fluxes parameterise the equator to pole heat exchange by synoptic scale eddies in the troposphere. In addition, the turbulent mixing affects a large spatial domain (mixing length) in a relatively short time, represented by the Rossby-radius, and extends into the near surface soil. Finally, Newtonian relaxation of the near surface soil or mixed layer ocean to their respective restoration temperatures, introduces a very long time scale, beyond which the system stabilises to a white noise response.

Time-fluctuations of the temperature anomalies (averaged over the mixing length) show a long-term memory regime, $\beta \sim 0.5$, which is close to observed or simulated data sets, considering their standard deviations. This may be interpreted as resonance affecting the domain-averaged temperatures fluctuations: They become correlated once periods (synoptic life cycle) are larger than the time scales required for diffusive mixing of the domain (mixing length of synoptic eddies). In other words, long-term memory sets in once the integral correlation scales in both space and time exceed those, which characterise diffusive mixing. This regime replaces the short-term memory scaling with $\beta \sim \frac{3}{2}$, which governs smaller length/time scales. Without a stabilising effect like Newtonian cooling, the long-term memory regime (or resonance) would expand to unlimited periods. The limitation, however, occurs once the white noise variability plateau is reached at the very low frequency end of the spectrum, which is dominated by the stabilising Newtonian or relaxation time scale.

The Fickian diffusion system with noisy fluxes and Newtonian cooling may also provide a toy model for testing new concepts of stochastic parameterisation. Such analyses have been suggested for the cumulus parameterisation problem (see Yano et al. 2001) and, almost certainly, for other scales. The existence of long-range memory in atmospheric data is relevant for significance tests, sample independence and extreme value statistics and, therefore, has an impact on the proper judgement of model induced climate change scenarios.

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