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A simple model for estimating the evaporation from a shallow water reservoir

By K. FRAEDRICH, A. BEHLAU, G. KERATH and G. WEBER, *Institut für Meteorologie, Freie Universität Berlin, 1 Berlin 33, FRG.*

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ABSTRACT

A simple model of the energy fluxes of a well-mixed water reservoir is presented based upon its energy and mass budget. The heating processes due to atmospheric-radiative and hydrological forcing are separated and parameterized in terms of two characteristic temperatures. Accordingly, two related time scales can be deduced which describe the energetical response of the reservoir on the two forcing mechanisms. Three different approximations for the latent heat fluxes are introduced. The model which includes the complete energy balance is applied to the Salton Sea (California) and shows good agreement with the observations.

1. Introduction

The energy balance of a water reservoir depends on the energy fluxes through the surface of the total water body and the storage within it. These energy fluxes can be separated into two parts: (i) the atmospheric-radiative processes due to an energy transfer which is not correlated with a flux of water substance, and (ii) the hydrological processes which are coupled with the water flux. (i) The latent heat flux (E) of evaporation, the sensible heat flux (H) and the net radiation (N) dominate the interaction of the water body with the environment at the air–water interface, the imbalance of which provides the main energy source (ΔS) for the hydrological processes within the reservoir:

$$\Delta S = N - (E + H) \tag{1.1}$$

(ii) The hydrological energy fluxes include the energy storage and those energy fluxes through the surface of the water body which are connected with a water flow (inflow, run-off, rainfall, water loss by evaporation).

In the following a simple model is developed to deduce the energy fluxes of a water reservoir from weather data of the atmospheric environment and, if the hydrological processes play an important role, from characteristic information of these. In dependence of the parameterization of the radiative (Section 2) and atmospheric (Section 3) energy transfer processes, a temperature and time scale can be defined (Section 4) characterizing the atmospheric-radiative forcing acting on the reservoir and its response time to it. Analogously, the parameterization of the hydrological processes leads to a characteristic temperature and time scale which describes the hydrological forcing of the reservoir and its response (Section 5). This forcing parameterized by characteristic temperatures and time scales leads to a simple differential equation describing the energy balance (temperature) of the water reservoir. As the evaporation is of great importance for almost all practical purposes, different parameterizations of this process are introduced (Section 3). An application is presented in Section 6 which is based upon observations from the Salton Sea.

2. Radiative fluxes

The radiative processes provide the main energy source for the latent and sensible heat transfer. The net radiative flux density N is given by the balance of the global radiation Q_s, its reflection Q_{sr}, the net incoming long-wave radiation $Q_a - Q_{ar}$ diminished by the long-wave emission from the water surface Q_L. In the following it is assumed that the radiative forcing R

$$R = (Q_s - Q_{sr}) + (Q_a - Q_{ar}) \tag{2.1}$$

is known from measurements or from a radiation climatonomy (Brunt formula, etc.). Thus, the net radiation is given by

$$N = R - Q_L \tag{2.2}$$

where long-wave emission Q_L can be approximated by a truncated Taylor series expansion

$$Q_L = \varepsilon\sigma T_o^4 \doteq \varepsilon\sigma \overset{*}{T}^4 + \varepsilon 4\sigma \overset{*}{T}^3 (T_o - \overset{*}{T}) \tag{2.3}$$

$\overset{*}{T}$ is an appropriate temperature close to the actual water surface temperature T_o which will be specified in Section 3. The emissivity of the water surface is denoted by ε.

3. Atmospheric fluxes

The sensible and latent heat fluxes are parameterized by the following bulk transfer formulas

$$E = L\alpha(q_o - q), \quad H = c_p\alpha(T_o - T) \tag{3.1}$$

with the turbulent transfer coefficient α (specified in Section 6), the specific humidity q and temperature T. The specific humidity difference can be expanded into a Taylor series which is truncated after the first derivative

$$q_o - q \doteq q_s - q + \left(\frac{dq_s}{dT}\right)_{\overset{*}{T}} (T_o - T) \tag{3.2}$$

$(q_s - q)$ is the saturation deficit of the air. This allows a combination of the energy budget (1.1) with the bulk transfer method (3.1) to obtain the Penman evaporation formula (Penman, 1949)

$$E_1 = \frac{N - \Delta S}{1 + B_1} + \frac{\alpha L(q_s - q)}{1 + B_1^{-1}} \tag{3.3a}$$

The coefficient

$$B_1 = \left(\frac{L}{c_p}\frac{dq_s}{dT}\right)^{-1}_{\overset{*}{T}} \tag{3.3b}$$

is inversely proportional to the first derivative of the Taylor series expansion and depends on the appropriate temperature $\overset{*}{T}$. This temperature is taken to be the same for both truncated Taylor series expansions (2.3, 3.2) and prescribed by the air temperature over the water surface. As such observations are hardly available (especially if new water reservoirs are to be planned) weather data must be used instead. Thus, $\overset{*}{T}$ may be replaced by an air temperature of a nearby weather station, i.e. $T = \overset{*}{T}$ in the following sections.

The Penman evaporation equation (3.3) explicitly shows the combination of the energetical and turbulent transfer processes (first and second term). Omitting the second (ventilation) term leads to the definition of an "equilibrium evaporation" which is supposed to occur after an infinitely long fetch over a saturated surface so that the saturation deficit of the air vanishes. As such a situation is rather unlikely to be of some representation, a formula similar to the equilibrium evaporation has been introduced which, according to observations is weighted by a factor $a = 1.26$ (Priestley & Taylor, 1972)

$$E_2 = a\,\frac{N - \Delta S}{1 + B_1} = \frac{N - \Delta S}{1 + B_2} \tag{3.4a}$$

Some general comments on the related Bowen ratio $B_2 = H_2/E_2$

$$B_2 = \frac{1}{a}(B_1 + 1 - a) \tag{3.4b}$$

of this second evaporation formula seem to be in order; it exhibits the "oasis effect" of hot radiation climates where, due to warm air advection, a downward (negative) sensible heat flux supplies additional energy for the evaporation process. According to eq. (3.4b) this occurs for $B_1 < (a - 1)$, i.e. for temperatures above 32° C (see Priestley, 1966).

In the following the subscripts (1 and 2) in (3.3a, b; 3.4a, b) will be omitted realizing that there are two approximations of the evaporation (Penman, 1949; Priestley & Taylor, 1972) to be considered simultaneously.

Once the surface temperature of the water reservoir is known (Section 5) a more conventional approach can be introduced

$$E_3 = \frac{N - \Delta S}{1 + B_3} \tag{3.5a}$$

However, this Bowen ratio

$$B_3 = \frac{c_p(T_o - T)}{L(q_o - q)} \tag{3.5b}$$

appears as the solution of an evaporation process under advective conditions assuming a well-mixed boundary layer over the water surface (Fraedrich, 1972; there the Bowen ratio B_3 appears as the ratio between the equilibrium evaporation (formula 20) and a similar formula for the equilibrium sensible heat flux, by replacing the specific humidity by the temperature). In this case T, q are explicitly prescribed by the temperature and humidity of the air in the environment (T_e, q_e in Fraedrich, 1972), and not over the water as it is requested by the traditional Bowen ratio concept (sometimes called Sverdrup method).

4. Equilibrium temperature

The parameterizations of the atmospheric and radiative fluxes (Sections 2 and 3) allow a calculation of the residual heat flux ΔS which changes the energy budget of the water reservoir. Thus, a combination of the evaporation formulas (3.3–3.4) with the energy balance (1.1), the net radiation (2.1–2.3) and the sensible heat flux (3.1) gives after some rearrangements

$$\Delta S = F - (T_o - T)\frac{cM}{\tau_a} \tag{4.1}$$

i.e. the residual heat flux ΔS is determined by a forcing function

$$F_1 = R - \varepsilon\sigma T^4 - L\alpha(q_s - q) \tag{4.2a}$$

$$F_2 = R - \varepsilon\sigma T^4 \tag{4.2b}$$

completely independent of the influence of the water reservoir, and a response of the water reservoir to this atmospheric-radiative forcing which is characterized by its adjustment time scale τ_a

$$\tau_a = \frac{cM}{c_p\,\alpha\,(1 + B^{-1}) + 4\varepsilon\sigma T^3} \tag{4.3}$$

where M is the mass of the water body per unit area, and c the specific heat of water.

An equilibrium temperature T_e can be introduced to replace the water surface temperature T_o (Edinger et al., 1968; Keijman, 1974) assuming the atmospheric-radiative energy source of the reservoir to vanish ($\Delta S = 0$ in 4.1; $T_o = T_e$):

$$T_e = T + F\cdot\frac{\tau_a}{cM} \tag{4.4}$$

The equilibrium temperature T_e is independent of the processes in the water reservoir which is obvious after simply replacing F and τ_a by (4.2, 4.3). With this definition (4.4) the energy source ΔS (4.1) can be rewritten as

$$\Delta S = cM\,\frac{T_e - T_o}{\tau_a} \tag{4.5}$$

which allows a better interpretation of τ_a and T_e: τ_a is the adjustment time of the water reservoir by the atmospheric-radiative processes towards T_e, i.e. it is the time scale by which the reservoir reacts upon this (thermal) forcing. The equilibrium temperature T_e represents the meteorological forcing which acts upon a reservoir and depends on sensible, latent and radiative heat transfer processes.

5. Energy balance of the water reservoir

A well-mixed reservoir is assumed for the following energy budget considerations, as it is most likely to be observed in tropical and even subtropical lakes:

$$c\left(\frac{\partial MT_o}{\partial t} - m_i\,T_i + m_o\,T_o\right) = \Delta S \tag{5.1}$$

The energy storage within the reservoir is balanced by hydrological processes which are connected with the inflow m_i and outflow m_o of the reservoir, and by the atmospheric-radiative heat source ΔS.

The mass outflow m_o (per unit area) consists of the surface and subsurface run off RO and the water leaving the reservoir by the evaporation process $L^{-1}E$:

$$m_o = RO + L^{-1}E \tag{5.2}$$

The related outflow of energy is characterized by the water temperature T_o. The mass inflow m_i (per unit area) is the sum of the surface and subsurface water accession with the rainfall being included. The related temperature T_i should be represented by a m_i-weighted average.

The continuity of water substance is

$$\frac{\partial M}{\partial t} = m_i - m_o \tag{5.3}$$

Combination of (5.3) and (5.1) and some rearrangements lead to the following energy equation:

$$cM\left(\frac{\partial T_o}{\partial t} + \frac{T_o - T_i}{\tau_h}\right) = \Delta S \tag{5.4}$$

where another time scale is defined:

$$\tau_h = \frac{cM}{cm_i} = \frac{M}{m_i} \tag{5.5}$$

τ_h characterizes the dynamics of the hydrological processes in terms of a mean residence time of a water molecule within the reservoir. Combination of (5.4) with the result (4.5) of Section 4 leads to a simple first order differential equation by which the energy balance (or the temperature T_o) of the water reservoir can be deduced analytically

$$\frac{\partial T_o}{\partial t} = \frac{T_i - T_o}{\tau_h} + \frac{T_e - T_o}{\tau_a} \tag{5.6}$$

This equation allows a simple interpretation because the two temperatures (T_i, T_e) and the two related time scales (τ_h, τ_a) corroborate the physical processes which act upon the reservoir: the atmospheric-radiative forcing (T_e) which the water reservoir follows according to the time scale τ_a; the hydrological dynamics, where the reservoir is influenced by the inflow temperature (T_i) with the characteristic response time τ_h. In this sense (5.6) represents the climatonomy of a water reservoir with its temperature T_o depending on these forcing functions parameterized by T_i, T_e and τ_h, τ_a. Under steady-state conditions (5.6) leads to a simple formula for the water temperature T_o:

$$T_o = \frac{\tau_a T_i + \tau_h T_e}{\tau_a + \tau_h}$$

To calculate the complete energy balance of the lake, the following input parameters are necessary which are prescribed by a time series and per finite increment Δt: (a) the temperature T and humidity of the air q, (b) the incoming radiation R (2.1), (c) the transfer coefficient α, (d) the mass M and mass flow m_i into the reservoir, (e) the inflow temperature T_i. Now τ_a (4.3), T_e (4.4), τ_h (5.5) can be calculated to deduce the surface temperature T_o (5.6) of the reservoir form an initial condition. This allows the energy source ΔS (4.5) and the net radiation N (2.2, 2.3) to be determined so that the evaporation E (3.3–3.4) and the sensible heat transfer H (3.1) can be obtained. As the input data is representative for the time increment Δt (e.g. a month) it is reasonable to solve (5.6) for

$$\hat{T}_o = \frac{1}{\Delta t}\int_0^{\Delta t} T_o\, dt \tag{5.7}$$

and use $\hat{T}_o$ (instead of T_o) to determine the energy flux N. The final T_o (5.6) of the time interval Δt is used to calculate the energy source ΔS with the fluxes E, H and also serves as the initial value for the following time step Δt. The new mass M of the water reservoir has to be derived from 5.2, 5.3 (if lake level variations are of considerable magnitude) to obtain the appropriate time scales τ_a, τ_h (4.3, 5.5).

It should be mentioned that for the condition

$$\tau_h \gg \tau_a$$

the hydrological process influencing the evaporation can be neglected so that the first term on the right-hand side of (5.6) may be omitted.

6. An application: The Salton Sea

The Salton Sea, California, was formed between 1905 and 1907 when the water was accidently diverted from the Colorado into the dry Salton Sink about 80 m below sea level. The surface area of this lake is about 800 km^2 with an average water depth of about 8 m. During 1961–62 the complete energy and water budgets of this reservoir have been observationally determined (Hughes, 1967) which serve as the basic data set to test our model.

The calculations are performed as outlined at the end of Section 5 with the input data R, T, q, T_i, m_i prescribed by monthly averages. The transfer coefficient

$$\alpha = \rho c_t u \tag{6.1}$$

mainly depends on the surface area of the water

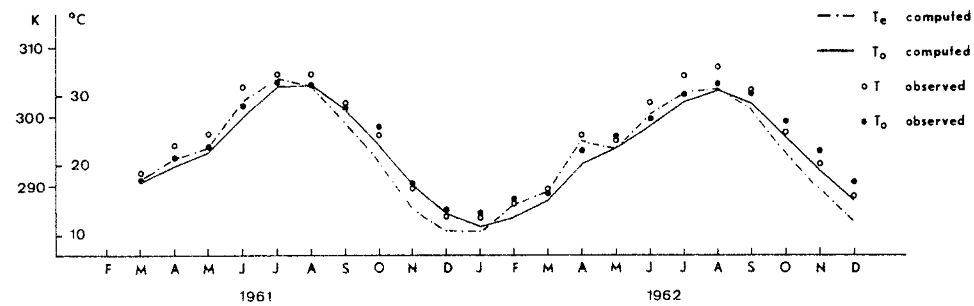

Fig. 1. Salton Sea temperatures; computed: equivalent temperature T_e (dashed-dotted line), water temperature T_0 (full line); observed: air temperature T (open circle), water temperature T_0 (full circle.).

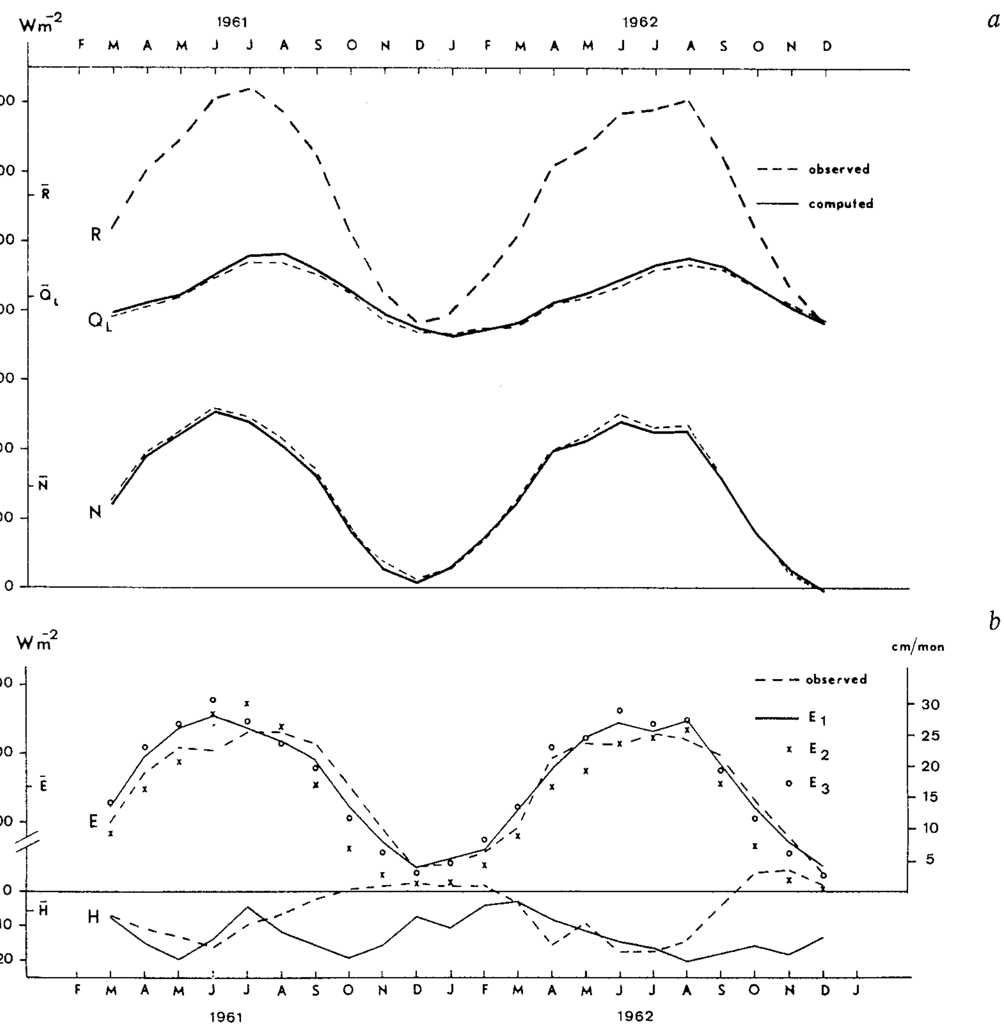

Fig. 2. Annual variation of the Salton Sea energy budget: (a) radiative forcing R, longwave emission Q_L, net radiation N. (b) evaporation E, sensible heat flux H (note the scale change for H!); (computed and observed: full and dashed lines). Observed averages are indicated on the left axis.

body ($c_t = 1.6 \cdot 10^{-3}$) and the windspeed u. All this information is extracted from the observations (Hughes, 1967) of which the incoming radiation R and the air temperature T are shown in Figs. 1 and 2.

The results of the energy budget and temperature computations presented in the following are mainly based on Penman's evaporation formula which appears to be the most realistic method of parameterization. The equilibrium temperature T_e as a measure of the driving atmospheric-radiative force (Fig. 1) for the water temperature T_o clearly outlines its characteristic features: it is above (below) the water temperature in spring and summer (fall and winter) thus leading to quite a realistic water temperature T_o and energy storage within the reservoir. The annual averages of the water, air and equilibrium temperature are almost the same: $\bar{T}_o \doteq \bar{T}_e \doteq \bar{T} \doteq 295.5°$ K, not a surprising result for reservoirs which are small enough not to regularly produce their own local atmospheric circulation system.

The energy fluxes are shown in Fig. 2. The results for the evaporation are presented using all three latent heat flux parameterizations, where E_3 is based on the water surface temperatures and energy fluxes determined by the Penman formula (E_1). It should be noted that the formulation E_2 (Priestley & Taylor, 1972) produces its own temperature and energy budget which is not shown here in order to confine ourselves to the most important results. The rms deviation of the three different latent heat flux parameterizations from the observed values are (in Watts m^{-2}) $\sigma_1 = 21.4$, $\sigma_2 = 42.2$, $\sigma_3 = 31.0$, i.e. $\sigma_1 < \sigma_3 < \sigma_2$ so that the Priestley–Taylor formulation does not appear the optimum for representing the annual variation but, this formulation has been calibrated to calm conditions. The annual averages are: $\bar{E}_1 = 156.0$, $\bar{E}_2 = 126.4$, $\bar{E}_3 = 157.3$ which can well be compared with the observed $\bar{E} = 150.9$ Watts m^{-2}.

Finally, it should be mentioned that the flux of sensible heat is not too realistically simulated which may be due to its small magnitude (note the change in scaling!) but also due to errors in the measurements (Hughes, 1967) and, of course, the shortcomings of this simple model.

7. Conclusions

This simple model describes the energy balance of a water reservoir in terms of two temperatures and time scales characterizing the energy fluxes as they appear separable into the hydrological and atmospheric-radiative processes, i.e. correlated and uncorrelated with a transport of water substance. Such a simplifying parametric description of the energy balances of the water reservoir allows some physical insight into the participating processes. Additionally, such a model can directly be applied to actual situations using simple meteorological (and, if necessary, hydrological) data and it leads to quite realistic results. However, it should be stressed that the situation of a well-mixed reservoir, as assumed here, is not always given. Further investigations will be needed to meet the more complicated natural conditions by some additional but simple parameterizations.

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9. List of symbols

Q_s, Q_{sr}	incoming, reflected short-wave radiation
Q_a, Q_{ar}	incoming, reflected long-wave radiation
Q_L	long-wave emission
N, R	net radiation, net incoming radiation
ε	emissivity
E, H	latent, sensible heat flux
L, c_p, c	latent heat of condensation, specific heat of air, water
$\alpha = \rho c_t u$	transfer coefficient
a	an empirical constant
B	Bowen ratio
ΔS	net energy source of the reservoir
F	forcing function
τ_a, τ_h	time scales of the reservoir
M, m	mass, mass flux (per unit area)
RO	run off (per unit area)
T, $\overset{*}{T}$	air temperatures (without subscript); see text

q	specific humidity
σ	Stefan Boltzmann constant

Indices

1, 2, 3	approximations of the evaporation process
o, i, e	water reservoir, inflow, equilibrium (temperature)
s	saturation
$\hat{}, \bar{}$	average over time step Δt, annual average

REFERENCES

Edinger, J. E., Duttweiler, D. W. & Geyer, J. C. 1968. The response of water temperatures to meteorological conditions. *Water Resour. Res. 4*, 1137–1143.

Fraedrich, K. 1972. On the evaporation from a lake in warm and dry environment. *Tellus 24*, 116–121.

Hughes, G. H. 1967. Analysis of techniques used to measure evaporation from Salton Sea, California. *Geological Survey Professional Paper 272-H*, 151–176.

Keijman, J. Q. 1974. The estimation of the energy balance of a lake from simple weather data. *Boundary Layer Meteor 7*, 399–407.

Penman, H. L. 1949. A general survey of meteorology in agriculture and an account of the physics of irrigation control. *Quart. J. Roy. Meteor. Soc. 75*, 293.

Priestley, C. H. B. 1966. The limitation of temperature by evaporation in hot climates. *Agr. Meteor. 3*, 241–246.

Priestley, C. H. B. & Taylor, R. J. 1972. On the assessment of surface heat flux and evaporation using large scale parameters. *Mon. Weath. Rev. 100*, 81–92.

ПРОСТАЯ МОДЕЛЬ ДЛЯ ОЦЕНКИ ИСПАРЕНИЯ С ПОВЕРХНОСТИ МЕЛКИХ ВОДНЫХ РЕЗЕРВУАРОВ.

Представлена простая модель потоков энергии от хорошо перемешанных водных резервуаров, основанная на балансе их энергии и массы. Процессы нагревания благодаря атмосферной радиации и гидрологии разделены и параметризованы в терминах двух характеристических температур. В соответствии с этим можно ввести два временных масштаба, описывающих энергетическую реакцию резервуара на два вида вынуждающих механизмов. Введены три различных аппроксимации для потоков скрытого тепла. Модель, включающая полный баланс энергии, применяется к Соленому Морю (Калифорния) и её выводы оказываются в хорошем согласии с данными наблюдений.