

## On the Parameterization of Cumulus Convection by Lateral Mixing and Compensating Subsidence. Part 1

K. FRAEDRICH

*Meteorologisches Institut der Universität Bonn, Bonn, West Germany*

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### ABSTRACT

Lateral mixing and compensating subsidence of penetrative convection are two different techniques of parameterizing the large-scale effects of deep cumuli. Both methods are jointly derived by averaging the conservative quantities: total and potential heat separately over the area of the hot towers and the environment. If separated, lateral mixing and compensating subsidence are equivalent with respect to their energetical effect integrated over the large-scale system. Due to this fact the fractional area covered by updrafts results in a simple relation.

### 1. Introduction

Several parameterization techniques are used in large-scale models to incorporate the effect of release of latent heat due to penetrative convection. Based on similar assumptions, these schemes mainly apply two different mechanisms, which physically describe the scale interaction between the ensemble of convective cells and the environment. These two mechanisms are: 1) the vertical mass flux relating the fluxes inside and outside a convective element [this (organized interaction) is actually an improved slice method, because compensating subsidence around the clouds warms the environment by adiabatic compression]; and 2) the lateral mixing of the total cloud (substance) into the environment.

The basic presumptions are that convection occurs in deep layers with conditionally unstable stratification (not necessarily) and a mean (large-scale and low-level) convergence. The single elements of the cumulus ensemble are alike, since only their total effect is thought important.

It is the purpose of this paper to show that the two mechanisms can be jointly derived by averaging suitable conservative extensive quantities separately over the cloud and cloud-free areas of a large-scale system, referring essentially to Arakawa (1971).

Separated, both mechanisms have the same energetical effect integrated over the large-scale system. For this condition a simple relation can be derived to determine the fractional cloud cover, which appears to be equivalent to Bjerknes' slice method.

### 2. Basic concept

The quantity (total heat)

$$H = c_p T + gz + Lq,$$

sometimes described as the static energy, is approximately conserved along air trajectories for inviscid and steady motion, neglecting the kinetic energy  $\frac{1}{2}v^2$  compared with the enthalpy  $c_p T$ , geopotential energy  $gz$ , and latent heat  $Lq$ . In this sense  $H$  is proportional to the equivalent potential temperature

$$\theta_E = \theta \exp \frac{Lq}{c_p T},$$

which is an approximation of the pseudo-adiabatic temperature. A similar relation holds for a dry adiabatic process; the quantity (potential heat)  $S = c_p T + gz$  is conserved and proportional to the potential temperature  $\theta$  with the above assumptions. The constants of proportionality for the differentials are  $T/\theta_E$ ,  $T/\theta$ .

In combination with mass and moisture conservation:

$$\nabla_h \cdot (\rho \mathbf{v}_h) + \frac{\partial \rho w}{\partial z} = 0, \quad (1)$$

$$\frac{d\rho q}{dt} = -C, \quad (2)$$

where  $\rho = \rho(z)$  only (anelastic approximation),  $C$  is the rate of condensation, and  $q$  the water vapor mixing ratio, the thermodynamical equations yield

$$\frac{d\rho H}{dt} = \frac{d\rho S}{dt} - LC = 0. \quad (3)$$

Integrating Eq. (3) over a horizontal area  $a = a(z, t)$  and applying the Leibniz rule for variable boundaries gives

$$\left( \frac{\partial \rho H_a}{\partial t} - H_R \frac{\partial \rho a}{\partial t} \right) + \left( \frac{\partial m_a H_a}{\partial z} - H_R \frac{\partial m_a}{\partial z} \right) = 0, \quad (4)$$

where the vertical mass transports at the lateral boundaries vanish. A top hat profile has been assumed with

$$\left. \begin{aligned} H_a a &= \int_a H df \\ m_a &= \int_a \rho w df = \rho(z) w_a a \end{aligned} \right\},$$

where  $H_R$  is the value at the boundary of the area, which will be determined in dependence of  $\partial \rho a / \partial t$  and  $\partial m_a / \partial z$ .

In the following  $a$  will be specified by the fractional (nondimensional) area  $\sigma$  of the updraft region of the cloud ensemble (subscript  $c$ ) and the area  $1-\sigma$  of its environment (subscript  $e$ ); thus, the large-scale averages yield

$$\left. \begin{aligned} \bar{m} &= m_e + m_c \\ \bar{H} &= (1-\sigma)H_e + \sigma H_c = \sigma(H_c - H_e) + H_e \end{aligned} \right\}. \quad (5)$$

### 3. Environment

As  $\sigma \ll 1$  and  $H_c - H_e \ll H_e$ , which is also valid for  $q$  and thus  $S$ , the energetical variables of the environment can be approximated by the large-scale averages. This, of course, is not valid for the mass fluxes.

There is no condensation occurring in the environment. Also, effective evaporation processes, which will be considered later, are (in this first approach) only of minor importance, which is true in the upper troposphere (above 850 mb). Thus,  $H_e$ , ( $\bar{H}$ ) can be separated into  $S_e$ , ( $\bar{S}$ ) and  $q_e$ , ( $\bar{q}$ ):

$$\left[ \frac{\partial}{\partial t} (\rho H_e (1-\sigma) - H_R \frac{\partial \rho (1-\sigma)}{\partial t}) \right] + \left[ \frac{\partial}{\partial z} (-H_e (\bar{m} - m_c) - H_R \frac{\partial (\bar{m} - m_c)}{\partial z}) \right] = 0.$$

Transforming the second term in brackets by the mass continuity equation (1) yields

$$\left[ \frac{\partial}{\partial t} (-\rho H_e (1-\sigma) + H_R \frac{\partial \rho \sigma}{\partial t}) \right] + \left[ H_R \frac{\partial m_c}{\partial z} - \frac{\partial H_e m_c}{\partial z} \right] = -\nabla_3 \cdot \overline{\rho \mathbf{v}_3} H_e, \quad (6)$$

where  $\nabla_3 \cdot \overline{\rho \mathbf{v}_3} = 0$  is the three-dimensional large-scale mass convergence. In this equation there appear exactly those parameters which are actually measured by the large-scale radiosonde network: the thermodynamical quantities of the environment and the wind velocities representing the large-scale features.

The separation into  $S$  and  $q$  and the approximations  $H_e \approx \bar{H}$ ,  $\sigma \ll 1$  lead to the large-scale equations for the

potential heat  $S$  (proportional to  $\theta$ ) and the moisture  $q$ :

$$\frac{\partial \rho \bar{S}}{\partial t} + \nabla_h \cdot \overline{\rho \mathbf{v}_h \bar{S}} + \frac{\partial \bar{S} \bar{m}}{\partial z} = Q_s, \quad (7)$$

$$\frac{\partial \rho \bar{q}}{\partial t} + \nabla_h \cdot \overline{\rho \mathbf{v}_h \bar{q}} + \frac{\partial \bar{q} \bar{m}}{\partial z} = Q_q. \quad (8)$$

The large-scale heat source due to penetrative convection is

$$Q_s = \left( \frac{\partial \bar{S} m_c}{\partial z} - S_R \frac{\partial m_c}{\partial z} \right) - S_R \frac{\partial \rho \sigma}{\partial t}, \quad (9)$$

and the moisture source

$$Q_q = \left( \frac{\partial \bar{q} m_c}{\partial z} - q_R \frac{\partial m_c}{\partial z} \right) - q_R \frac{\partial \rho \sigma}{\partial t}. \quad (10)$$

The boundary values  $H_R$ ,  $S_R$  and  $q_R$ , and the effect of the individual parts of the diabatic large-scale heating including evaporation, can be better interpreted with reference to the related model of the cloud ensemble. The first term describes the effect of cumulus clouds if there is no accumulative storage of energy within them; the second indicates the influence of this storage term.

### 4. Cloud ensemble

The fundamental (area-averaged) equation is

$$\left[ \frac{\partial}{\partial t} (\rho H_c \sigma) - H_R \frac{\partial \rho \sigma}{\partial t} \right] + \left[ \frac{\partial H_c m_c}{\partial z} - H_R \frac{\partial m_c}{\partial z} \right] = 0. \quad (11)$$

#### 1) ENTRAINMENT

If we consider an entrainment (inflow:  $\partial m_c / \partial z > 0$ ) and a detrainment layer (outflow:  $\partial m_c / \partial z < 0$ ), the boundary values are simply determined:

$$H_R = \begin{cases} H_e, & \text{in the inflow region} \\ H_c, & \text{in the outflow region} \end{cases}$$

Observations indicate that in a statistical sense the cloud top heights do not deviate from the height of a local equilibrium which is defined by corresponding thermodynamical parameters of the environment and the clouds (Simpson *et al.*, 1965). One can conclude that the outflow (below cloud top) also occurs in a layer of equilibrium with the environment. Thus, the environmental variables ( $H_e$ ,  $S_e$ ,  $q_e$ ) can be taken as the boundary values ( $H_R$ ,  $S_R$ ,  $q_R$ ) correlated with both the inflow and outflow layer.

#### 2) STORAGE

As  $\sigma$  is assumed to be sufficiently smaller than 1, both terms in the first brackets of (11) would seem to be

negligible. But this is correct for the first term only, since  $\partial\rho\sigma/\partial t$  is actually composed by the sum of the production rate ( $p$ ) of cloudy air and its dissipation rate ( $d$ ):

$$\frac{\partial\rho\sigma}{\partial t} = \left(\frac{\partial\rho\sigma}{\partial t}\right)_p + \left(\frac{\partial\rho\sigma}{\partial t}\right)_d \approx 0, \quad (12)$$

i.e., a small difference of probably two large quantities which can be neglected only if  $(\partial\rho\sigma/\partial t)_p$  and  $(\partial\rho\sigma/\partial t)_d$  are not correlated with different boundary values  $H_R$ ,  $S_R$ ,  $q_R$  as weighting factors.

But during the growing phase cloud mass is injected into environmental air. Thus, the corresponding boundary value correlated with the cloud mass production must represent the environment:

$$H_e \left(\frac{\partial\rho\sigma}{\partial t}\right)_p.$$

In the decaying stage the opposite is true and the cloud mass is laterally mixed into the environment; thus, the correlated value at the boundary represents the cloudy air:

$$H_c \left(\frac{\partial\rho\sigma}{\partial t}\right)_d = -H_c \left(\frac{\partial\rho\sigma}{\partial t}\right)_p,$$

where  $(\partial\rho\sigma/\partial t)_p \approx -(\partial\rho\sigma/\partial t)_d$  because  $\sigma \ll 1$ .

Taking into account considerations 1) and 2), the resulting equation for the cloud system becomes

$$\left(\frac{\partial\rho\sigma}{\partial t}\right)_p (H_e - H_c) = \frac{\partial m_c H_c}{\partial z} - H_e \frac{\partial m_c}{\partial z}, \quad (13)$$

where, on separating  $H$  into  $S$  and  $q$ , we have

$$\frac{\partial m_c S_c}{\partial z} - S_e \frac{\partial m_c}{\partial z} - LC = \left(\frac{\partial\rho\sigma}{\partial t}\right)_p (S_e - S_c), \quad (13a)$$

$$\frac{\partial m_c q_c}{\partial z} - q_e \frac{\partial m_c}{\partial z} + C = \left(\frac{\partial\rho\sigma}{\partial t}\right)_p (q_e - q_c). \quad (13b)$$

The production of the cloud property  $(\partial\rho\sigma/\partial t)_p (S_e - S_c)$  is determined by condensation ( $LC$ ), lateral entrainment [ $S_e(\partial m_c/\partial z)$ ], and vertical divergence of the potential heat flux  $(\partial m_c S_c/\partial z)$ . A similar description is valid for the moisture field of the cloud mass.

Integration of (13) over the total layer depth  $Z$  of the cloud ensemble gives

$$\left(\frac{\partial\sigma}{\partial t}\right)_p = \frac{\int_0^{\Delta Z} \frac{\partial m_c}{\partial z} H_e dz + (m_c H_c)_0}{\int_0^{\Delta Z} \rho (H_e - H_c) dz}. \quad (14)$$

This expression is very similar to Kuo's method of parameterization [Kuo, 1965, Eq. (3.10)]. The denominator describes the total amount of energy acquired for creating the cloud mass in the column, and the numerator the total rate of accession of energy to the column, which Kuo prescribes by the large-scale accession of water vapor. If one does not account for the vertical mass flux to produce the effects of the latent heat release, i.e., if

$$\frac{\partial m_c S_c}{\partial z} - S_e \frac{\partial m_c}{\partial z} \ll (S_e - S_c) \left(\frac{\partial\rho\sigma}{\partial t}\right)_p, \quad (15)$$

Eqs. (13) and (14) lead exactly to Kuo's parameterization technique. If the production rate of cloud mass  $(\partial\rho\sigma/\partial t)_p$  is neglected compared with the other terms, i.e., there is no accumulation of heat and moisture in the cloud ensemble, Eq. (13) leads to the cloud model developed by Arakawa (1971), Yanai (1971), and others.

The problem remains to combine the total cloud mass production  $(\partial\rho\sigma/\partial t)_p$  with the vertical mass flux  $m_c$  of the clouds, and to determine either one depending on the large-scale parameters.

Observations show that the average lifetime of individual cumulonimbus cells is of the order of 1 hr, half of which is spent in growing and half in dissipation, if one takes the cloud top heights as an indicator for the state of development (Cruz, 1973). The relatively small time scale (half-life) of individual hot towers in relation to the large-scale changes (roughly the time for a cloud parcel to ascent from cloud base to top) allows some simplifying assumptions for the convective cells. The deep cumuli embedded in a large-scale system grow from cloud base to their maximum height (within  $\frac{1}{2}$  hr) and then decay. However, they do not grow any further on their own, as in the case of squall lines, for example, which continuously transport mass upward and build a large outflow system. Thus, the vertical mass flux at the cloud base,  $m_{c0}$ , is used for creating new clouds after others have reached maturity (i.e., maximum height). This is in some agreement with an explanation of the dynamical motion of tropical cumulonimbi which generally travel faster than the vertically averaged tropospheric wind would allow, i.e., as shown by Cruz (1973) the radar-observed progression of "one" hot tower is a sequence of growing deep cumuli one ahead of the other in the direction of the cloud motion.

Lateral advection into the cloud system is entrainment and the compensating subsidence affects the environment only during the growing period. Thus, the production of cloud mass affecting the large-scale system can be determined by

$$\left(\frac{\partial\rho\sigma}{\partial t}\right)_p = \frac{m_{c0}}{\Delta Z} + \frac{\partial m_c}{\partial z}, \quad (16)$$

where  $\Delta Z$  is the scale height of the clouds, i.e., the maxi-

mum cloud heights. With the generally used form of the entrainment function

$$\frac{1}{m_c} \frac{\partial m_c}{\partial z} = \frac{2\alpha}{R} \rightarrow m_c = m_{c0} \exp\left(-\frac{2\alpha}{R} Z\right),$$

( $2\alpha=0.2$ , where  $R$  is the radius of the individual cumulus cells, which are assumed to be alike). It follows that

$$\left(\frac{\partial \rho \sigma}{\partial t}\right)_p = m_{c0} \left( \frac{1}{\Delta Z} + \frac{2\alpha}{R} \exp\left(-\frac{2\alpha}{R} Z\right) \right)$$

or

$$\approx \frac{m_{c0}}{\Delta Z} \left( 1 + \exp\left(-\frac{z}{\Delta Z}\right) \right),$$

if the entrainment is scaled by  $1/\Delta Z$ .

### 5. The large-scale heat and moisture source

The large-scale heat and moisture source due to penetrative convection can be specified by using the following arguments:

1) Neglecting the second (mixing) terms of the large-scale heat (9) and moisture sources (10) leads to the slice (Arakawa's) method of parameterizing convection:

$$\begin{aligned} Q_s &= m_c \frac{\partial S_e}{\partial z}, \\ &\approx m_c \frac{\partial \bar{S}}{\partial z}, \\ Q_q &\approx m_c \frac{\partial \bar{q}}{\partial z}. \end{aligned}$$

It appears reasonable that  $(q_c - q_e)(\partial m_c / \partial z)$  and  $(S_c - S_e)(\partial m_c / \partial z)$  (correction terms in the outflow layer) can be neglected compared with  $[m_c(\partial q_e / \partial z)]$  or  $[m_c(\partial S_e / \partial z)]$ . This provides an additional argument for the validity of the assumption  $q_e \approx q_c$  and  $\theta_c \approx \theta_e$  in the detrainment layer.

2) Neglecting the first terms in (9) and (10) leads to the mixing (Kuo's) technique to include convection in large-scale modeling:

$$\begin{aligned} Q_s &\approx \left(\frac{\partial \rho \sigma}{\partial t}\right)_p c_p (T_c - \bar{T}), \\ Q_q &\approx \left(\frac{\partial \rho \sigma}{\partial t}\right)_p (q_c - \bar{q}), \end{aligned}$$

We still use the general form of  $(\partial \sigma / \partial t)_p$  in Eq. (14), however.

These two processes can be combined in the following sense to include the life cycle of the individual hot towers and its effect on the environment.

- (i) During the growing (and mature) phase the diabatic heating due to condensation and convective heat transport is felt by the large-scale system as adiabatic warming due to compensating subsidence between the clouds.
- (ii) During the decaying period the diabatic heating of the large-scale system due to penetrative convection is performed by mixing of the cloudy air of the (collapsing) hot towers into the environment. This occurs as the clouds are no longer buoyant (i.e., the virtual temperatures of the environment and the clouds are alike). The result is that water droplets stored in the clouds evaporate, and saturated or unsaturated downdrafts develop and cool the environment by evaporation.

Making use of (15), the production of latent heat by condensation yields a simple form

$$LC = \left(\frac{\partial \rho \sigma}{\partial t}\right)_p c_p (T_c - T_e).$$

Thus, the evaporational cooling of the environment can be taken into account in a manner analogous to the warming by mixing (Kuo, 1965):

$$\left. \begin{aligned} Q_s &= m_c \frac{\partial \bar{S}}{\partial z} - \alpha \left(\frac{\partial \rho \sigma}{\partial t}\right)_p c_p (T_c - \bar{T}) \\ Q_q &= m_c \frac{\partial \bar{q}}{\partial z} - \left(\frac{\partial \rho \sigma}{\partial t}\right)_p \left[ (q_c - \bar{q}) + \alpha \frac{c_p}{L} (T_c - \bar{T}) \right] \end{aligned} \right\} \quad (17)$$

The weighting factor  $\alpha$  is the ratio of evaporation to condensation, or  $\beta = 1 - \alpha$ , where  $\beta$  is the rainfall/condensation ratio. These (probably) height-dependent factors  $\alpha$  or  $\beta$  may be determined by the internal water cycle of the hot towers and are related to the parameters of a larger scale (three-dimensional wind shear, cloud top heights, saturation deficit of the environment, etc.). They can be interpreted as a correction to yield a physically adequate expression for the boundary values  $S_R, q_R$ .

Arakawa (1971) describes the details how the vertical mass flux of the cloud system can be determined from the large-scale parameters by a physical consideration on the bouyancy of the cumulus clouds (Yanai, 1971). Applying the relation (16), this offers a possible closure for the parameterization scheme (17), or Kuo's method relating the cloud mass production to the large-scale moisture convergence.

### 6. Example

The process of lateral mixing due to the accumulation of heat and moisture in the clouds tends to produce a saturated environment with the related (moist-adiabatic) temperature distribution of the cloud ensemble.

The mechanism of compensating subsidence tends to produce a dry and dry-adiabatically stratified environment [see Eqs. (7) and (8)]. The two processes coexist and contribute to the actual change in the environment.

If separated, both methods have the same energetical effect integrated over the large-scale system, because the energy produced by penetrative convection due to the release of latent heat is the same. Based on this assumption some diagnostic relations can be derived.

The lateral mixing process affects the total layer of the large-scale system, which leads to a constraint on the environmental layer depth influenced by compensating subsidence:

$$\int_z \left( \frac{\partial \rho (1-\sigma)}{\partial t} \right)_p c_p (T_e - T_c) dz = \int_z m_c \frac{\partial S_e}{\partial z} dz. \quad (18)$$

In the following an undiluted (i.e., vertically constant) mass transport  $m_c = m_{c0}$  is assumed. Consequently, the moist adiabat describes the thermodynamics of the convective ensemble. The cloud base ( $b$ ) is given by the lifting condensation level. The cloud top ( $t$ ) is the level where the moist adiabat through the cloud base meets the environmental temperature profile. During the cumulonimbus cell time scale  $\Delta t \approx \frac{1}{2}$  hr, the total cloud mass produced for the lateral mixing process is

$$\begin{aligned} m_c &= \frac{1}{\Delta t} \int_0^\sigma \int_0^{\Delta Z} \rho dz df \\ &= \frac{1}{\Delta t} \int_0^\sigma \int_0^{\Delta P} \frac{dp}{g} df = \int_0^{\Delta Z} \left( \frac{\partial \rho \sigma}{\partial t} \right)_p dz, \end{aligned} \quad (19)$$

where  $\Delta P = P_b - P_t$  and  $\Delta Z = Z_t - Z_b$  is the depth (scale) of the cloud layer. This is equivalent to the environmental subsidence which may be interpreted as a mass production into the environment:

$$m_c = \frac{1}{\Delta t} \int_0^{1-\sigma} \int_{\Delta Z - \delta z}^{\Delta Z} \rho dz df = \frac{1}{\Delta t} \int_0^{1-\sigma} \int_0^{\delta P} \frac{dp}{g} df. \quad (20)$$

A simple relation results from (19) and (20) for the environmental layer depth  $\delta P$  affected by subsidence, i.e.,

$$\sigma \Delta P = (1-\sigma) \delta P \quad \text{or} \quad \sigma / (1-\sigma) = \delta P / \Delta P. \quad (21)$$

The total amount of potential heat which is produced by the hot towers and which affects the large-scale system due to lateral mixing is

$$c_p \delta T = \frac{c_p}{\Delta P} \int_{P_t}^{P_b} (T_c - T_e) dP. \quad (22)$$

The same value must be produced by compensating subsidence. Thus, with a constant vertical lapse rate of the environmental potential heat ( $\partial S_e / \partial z = \text{constant}$ ), Eqs. (18)–(20) and the assumption  $m_c = m_{c0} = \text{constant}$

lead to the relation

$$c_p \delta T = \frac{\Delta S_e}{\Delta P} \delta P, \quad (23)$$

where  $c_p^{-1} \Delta S_e \approx \Delta \theta_e$ , the potential temperature difference of the environment between cloud top and cloud base. Combination of (21), (23) leads to some simple diagnostic relations:

$$\frac{\sigma}{1-\sigma} = \frac{\delta P}{\Delta P} = \frac{\delta T}{\Delta \theta_e} \approx \sigma \rightarrow \frac{\gamma - \gamma_m}{\gamma_a - \gamma}, \quad (24)$$

where  $\gamma$ ,  $\gamma_a$ ,  $\gamma_m$  are the temperature gradients: geometric, dry adiabatic, and moist adiabatic (Bjerknes, 1938; Cressman, 1946).

The following values are based on the mean tropical atmosphere for the hurricane season:

$$\delta T \approx 2K, \quad \Delta \theta_e \approx 60K, \quad \Delta P \approx 800 \text{ mb.}$$

Thus, the fractional area covered by hot towers is approximately

$$\sigma \approx 3-4\%.$$

The layer depth effected by subsidence within the half-life of hot towers is

$$\delta P \approx 25 \text{ mb.}$$

During a 10-hr ( $\approx 20 \times \Delta t$ ) passage of a synoptic system, the layer depth of about 500 mb is warmed by  $\delta T$  due to subsidence of the environmental column.

According to these results [Eq. (24)] the fractional area covered by hot towers decreases as the environment approaches a moist-adiabatic state, which should be verified by cloud observations. This occurs because the heat production by lateral mixing only [Eq. (22)] is estimated to be zero, when  $T_e$  approaches  $T_c$  (Kuo, 1965).

## 7. Conclusion

The main purpose of the study was to develop a theoretical formalism for the parameterization of deep convection, in which empirical and analytical information and the possible influence of the cumulonimbus life cycle can be incorporated. A more detailed specification is needed on how the coexisting processes of lateral mixing (detrainment) and compensating subsidence participate in changing the environment. This can be developed if the physical properties of the hot towers are known. The assumption that an ensemble of individual buoyant bubbles represents the effects of cumulus convection [Ooyama, 1971, Eq. (29)] yields similar results for the large-scale equations.

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