On the evaporation from a lake in warm and dry environment

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ABSTRACT

Evaporation suppressed by an inversion can occur over water reservoirs, when air is advected from a dry and warm environment. A proposed model of this process is based on the conservation of heat and moisture, and on the energy balance at the water surface. Under steady state conditions the inversion height, and the turbulent heat and water vapor fluxes are analytically determined. They depend on the travel distance of the air over the water surface, on simple turbulence parameters, on the net radiation at the water surface, and the temperature and moisture distribution over the land. An example is presented for the case of a well mixed environment.

Introduction

A shallow stably stratified layer (inversion layer) is produced by a negative (downward) heat flux over a water surface, when the air is blowing from the warmer land. Additionally, if the advected air is relatively dry, evaporational cooling amplifies the stable layer.

Such a shallow inversion layer over the water surface will reduce the evaporation of the reservoir. This will mainly occur during all times in spring and summer and during the day time in winter. It is the purpose of this paper to develop a simple model of this evaporation process.

Many further effects can participate:

(*a*) The air accelerates due to differences in the surface stress between land and lake.

(*b*) Secondary flows of thermal origin develop by the temperature differences between the land and the lake and by a pressure increase, because the inversion layer cools.

(*c*) The divergence of the (long wave) radiative flux in the inversion layer due to the moisture increase tends to grow the inversion strength and height.

(*d*) Interaction between the air and the water varies (decreases) the water surface temperature along the air trajectory. This can lead to an inversion, even if the air temperature of the environment is equal or less compared with the temperature of the water surface at the shore.

These effects are assumed to have only little influence, and they are not taken into account for this first analysis. Measurements that indicate the existence and the consequences of an inversion are rare. The hydrological and energy balance data of e.g. Lake Mead (Nevada) show a negative sensible heatflux between March and October (Harbeck et al., 1958). Furthermore it is well known that the evaporation from a pan can far exceed the evaporation from a water reservoir. One important cause probably is the inversion, as it increasingly suppresses the evaporation—increasing with the travel distance of the air over the reservoir ("vapour blanket"). The modification of convectively well mixed warm air blowing from the land over a cool sea surface has been studied near Massachusetts Bay (Craig, 1946). Micrometeorological experiments show this effect over an irrigated grass field and how the microclimate is modified on a relatively large scale (Dyer & Crawford, 1965; Rider et al., 1963).

Modelling

To simply illustrate the mechanism of this process the Boussinesq approximation is used, which is valid up to heights smaller than

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the scale height of the atmosphere. Then the potential temperature θ replaces the temperature as a thermal variable (Ogura & Phillips, 1962):

$$\frac{d\theta}{dt} = -\frac{\partial}{\partial z}\overline{(w'\theta')}. \quad (1)$$

Similar is the conservative equation of the specific humidity q:

$$\frac{dq}{dt} = -\frac{\partial}{\partial z}\overline{(w'q')}, \quad (2)$$

i.e. the total change of potential temperature and specific humidity is produced by the vertical divergence of the turbulent θ-flux: $\overline{w'\theta'}$ and q-flux: $\overline{w'q'}$. The process will be confined to a steady state with a negligible mean vertical motion and a vertically constant horizontal velocity u representing the near surface wind. Thus the total derivative is approximated by

$$\frac{d}{dt} \doteq u\frac{\partial}{\partial x}, \quad (3)$$

where x is the travel distance of the air in wind direction offshore from the land–lake boundary (fetch).

An average temperature $\bar{\theta}$ of a column with the variable height h is defined by

$$\bar{\theta} = \frac{1}{h}\int_0^h \theta\, dz.$$

The total change of $\bar{\theta}$ becomes:

$$\frac{d\bar{\theta}}{dt} = -\frac{1}{h^2}\frac{dh}{dt}\int_0^h \theta\, dz + \frac{1}{h}\left[\frac{dh}{dt}\theta(h) + \int_0^h \frac{d\theta}{dt}dz\right]$$

and using eq. (1):

$$\frac{d\bar{\theta}}{dt} = \frac{1}{h}\frac{dh}{dt}(\theta(h) - \bar{\theta}) - \frac{1}{h}(\overline{(w'\theta')}_h - \overline{(w'\theta')}_0), \quad (4)$$

where $\theta(h)$ is the temperature at the top of the column. The analogue equation is derived for a mean specific humidity:

$$\bar{q} = \frac{1}{h}\int_0^h q\, dz,$$

$$\frac{d\bar{q}}{dt} = \frac{1}{h}\frac{dh}{dt}(q(h) - \bar{q}) - \frac{1}{h}(\overline{(w'q')}_h - \overline{(w'q')}_0). \quad (5)$$

Both $\bar{\theta}$ and $\bar{q}$ can as well be interpreted as variables of a well mixed column. In the evaporation process to be modelled the top of the column is the top of the inversion layer, where turbulent fluxes of heat $H = c_p \varrho \overline{w'\theta'}$ and water vapour $E = \varrho \overline{w'q'}$ become negligibly small:

$$\overline{(w'\theta')}_h \sim 0, \quad \overline{(w'q')}_h \sim 0. \quad (6)$$

The turbulent fluxes of heat H_0 and water vapor E_0 at the evaporating surface are commonly parametrized. They are proportional to the wind u, to the transfer coefficient c_t, which is to be the same for heat and water vapor, and to the humidity and temperature difference between the water surface and the layer under the top of the inversion:

$$H_0 = c_p \varrho \overline{(w'\theta')}_0 = c_p \varrho c_t(\theta_0 - \bar{\theta}),$$

$$E_0 = \varrho \overline{(w'q')}_0 = \varrho c_t(q_0 - \bar{q}). \quad (7)$$

The density ϱ remains constant; for neutral stratification the transfer coefficient c_t has been found to vary with wind speed as follows (Roll, 1965):

$$c_t = (1.10 + 0.04\, u)\, 10^{-3};\ u \text{ in m sec}^{-1}$$

This form of parametrization of the surface fluxes is valid at a given (instrumental) height and under near neutral conditions. It can be justified by the (state of the) inversion layer, which has been interpreted to be well mixed. With this the balance of the energy fluxes at the water surface becomes the closure of the equations describing the evaporation process:

$$Q = H_0 + LE_0, \quad (8)$$

Q: Net radiation.

The evaporation problem can be treated differently as diffussion of water vapor from a finite area (Sutton, 1953). This approach appears to be limited, because it is non-energetic. But the moisture flux is directly connected with the heat balance. Thus it is necessary to solve simultaneously for the humidity and temperature field and to link both fields by the energy balance. The equations (4), (5),

(8) with the approximations (3), (6), (7) are the basic system to be solved for the horizontal variation of the inversion height $h(x)$ and humidity $q(x)$, and consequently of the turbulent heat and water vapor fluxes over the reservoir; i.e. this process has been reduced to an one-dimensional problem:

$$\frac{\partial}{\partial x}(h\bar{\theta}) = c_t(\theta_0 - \bar{\theta}) + \theta(h)\frac{\partial h}{\partial x}, \tag{9}$$

$$\frac{\partial}{\partial x}(h\bar{q}) = c_t(q_0 - \bar{q}) + q(h)\frac{\partial h}{\partial x}, \tag{10}$$

$$Q = c_p \varrho c_t u \left[(\theta_0 - \bar{\theta}) + \frac{L}{c_p}(q_0 - \bar{q})\right]. \tag{11}$$

Known quantities are the net radiation at the water surface Q, the water surface temperature θ_0 with the saturation humidity $q_0 = q_s(\theta_0)$. They are assumed to be constant so that the mechanism of evaporation is determined by the partition between the different flux components only. Furthermore the temperature $\theta(h)$ and humidity $q(h)$ at the top of the inversion are given by the vertical temperature and humidity distribution of the undisturbed upwind environment. Also the wind velocity u is prescribed.

Solution

For simplicity the environmental field of temperature and humidity, which determines the undisturbed thermal and moisture distribution at the inversion top, is prescribed to have linear vertical gradients:

$$\theta(h) = \theta_e - \Gamma h, \; q(h) = q_e - \Gamma_q h,$$

$$\left(\Gamma = -\frac{\partial \theta}{\partial z}; \; \Gamma > 0 \text{ unstable}; \; \Gamma < 0 \text{ stable}; \; \Gamma = 0 \text{ neutral}\right).$$

A log-profile stratification will not appreciably change the results in principle but would increase the numerical effort (Taylor, 1971).

Adding eq. (9) and (10)$\cdot L/c_p$, and applying the heat balance equation (11), a nonlinear differential equation for the inversion height $h(x)$ can be deduced.

$$h\frac{\partial h}{\partial x}\left(\Gamma + \frac{L}{c_p}\Gamma_q\right) + \frac{\partial h}{\partial x}\left[(\theta_0 - \theta_e) + \frac{L}{c_p}(q_0 - q_e) - \frac{Q}{c_p \varrho c_t u}\right] = \frac{Q}{c_p \varrho u}. \tag{12}$$

The resulting inversion height $h(x)$ becomes a square root function of the travel distance x over the reservoir:

$$h(x) = -\frac{B}{A} + \sqrt{\left(\frac{B}{A} + z_0\right)^2 + 2c_t Bx} \tag{13}$$

where $A = Q/\left[c_p \varrho c_t u\left(\theta_0 - \theta_e + \frac{L}{c_p}(q_0 - q_e)\right) - Q\right]$,

$B = \dfrac{Q/c_p \varrho c_t u}{\Gamma + (L/c_p)\Gamma_q}$, and z_0:

inversion height at $x = 0$, where the air trajectory over the lake starts; z_0 may also be interpreted as a roughness length at the land–lake interface.

Two important constraints have to be considered: As a complex or negative inversion height $h(x)$ is physically intractable, the described process will only occur under the condition $B > 0$: $Q > 0$ or < 0 at the water surface must coincide with $\Gamma + L/c_p \Gamma_q > 0$ or < 0 of the environment, which is reasonable.

The top of the inversion $h(x)$ cannot increase to infinity but is limited by the lifting condensation level, where stratiform clouds will develop.

Introducting $h(x)$ into eq. (9) yields the horizontal distribution of the mean humidity under the inversion:

$$\bar{q}(x) = \exp\frac{z_0 - h(x)}{B}\left(\frac{z_0}{h(x)}\right)^{1+A^{-1}} \times \left\{[q_e - q_0 + \Gamma_q B(1+A)^{-1}]\int_0^{h(x)} \exp\frac{h' - z_0}{B}\left(\frac{h'}{z_0}\right)^{1/A} \times \frac{dh'}{z_0} + \bar{q}_0\right\} + (q_0 - \Gamma B)\left\{1 - \exp\frac{z_0 - h}{B}\left(\frac{z_0}{h}\right)^{1+A^{-1}}\right\}. \tag{14}$$

$\bar{q}_0$ is the boundary value at $x = 0$, which is given by the humidity close to the land surface q_e.

If the length of the air trajectory over the water reservoir is large enough so that $z_0 << h$, the mean humidity under the inversion approaches (in the following written as $\rightarrow$) a constant value:

$$\bar{q} \rightarrow q_0 - \Gamma_q B.$$

The local evaporation at the water surface depending on the fetch can easily be determined:

$$E_0(x) = c_t u \varrho (q_0 - \bar{q}(x)) \rightarrow c_t u \varrho \Gamma_q B. \tag{15}$$

The total evaporation (per unit area) after the travel distance x is given by integration of eq. (9) ($z_0 \ll h$):

$$\begin{aligned}\tilde{E}_0(x) &= \frac{1}{x}\int_0^x E_0(x')dx' \\ &\doteq \varrho u \frac{h(x)}{x}\left[\bar{q}(x) - \left(q_e - \Gamma_q \frac{h(x)}{2}\right)\right] \\ &\rightarrow \varrho u \frac{h(x)}{x}\left[q_0 - q_e - \Gamma_q\left(B - \frac{h(x)}{2}\right)\right].\end{aligned} \tag{16}$$

For large x the local and total evaporation approach the same value (15). If only the humidity field of the environment is well mixed: $\Gamma_q = 0$ but $\Gamma \neq 0$:

$$\tilde{E}_0(x) \rightarrow \varrho u \frac{h(x)}{x} q_0 - q_e.$$

Due to the similarity of eq. (9) and (10) the replacement of $\bar{q}(x)$, $\bar{q}_0 = \bar{q}(x=0)$, q_e, q_0, Γ_q, and E_0 by $\bar{\theta}(x)$, $\bar{\theta}_0 = \bar{\theta}(x=0)$, θ_e, θ_0, Γ, and H_0 leads to the related expressions of the temperature field: the mean temperature under the inversion and the turbulent flux of heat at the water surface.

The new formulae for the evaporation and the turbulent heat flux (per unit area) (15, 16) depend on the length of the air trajectory over the water reservoir. Further important parameters are the net radiation at the water surface and the thermal and moisture distribution of the environment.

This model is valid, if the turbulent heat flux is negative at the land–lake boundary: $\theta_0 < \theta_e$, and remains so: $\Gamma \cdot B < 0$ (15). An example will be presented for the first of the two special cases which are discussed in the following.

Case 1

$$\Gamma = \frac{L}{c_p}\Gamma_q = 0, \text{ i.e. } B^{-1} = 0.$$

The environment is totally mixed. This assumption is useful for a first approach and a climatological estimation. In this case the inversion height becomes a linear function of the travel distance:

$$\frac{\partial h}{\partial x} = c_t A. \tag{17}$$

The slope of the inversion height must be positive, $A > 0$:

$$h(x) = c_t A x + z_0.$$

The humidity (and temperature) field under the inversion depending on the fetch is:

$$\bar{q}(x) = \left(1 - \frac{z_0}{h(x)}^{1+A^{-1}}\right)\left(\frac{q_0}{A} + q_e\right)(1 + A^{-1})^{-1} + \bar{q}_0\left(\frac{z_0}{h(x)}\right)^{1+A^{-1}}, \quad \bar{q}_0 = \bar{q}(x=0). \tag{18}$$

and for long travel distances ($z_0 < < h(x)$):

$$\bar{q} \rightarrow \left(\frac{q_0}{A} + q_e\right)(1 + A^{-1})^{-1}. \tag{19}$$

It follows that the local and total evaporation can easily be deduced from eqs. (15) and (16):

$$\left.\begin{aligned}E_0(x) &= \varrho c_t u (q_0 - \bar{q}(x)) \\ \tilde{E}_0(x) &= \varrho c_t u (q_0 - \bar{\bar{q}}(x)) \\ &= \varrho u \frac{h(x)}{x}(\bar{q}(x) - q_e)\end{aligned}\right\} \rightarrow \varrho c_t u (q_0 - q_e)(1 + A^{-1})^{-1}. \tag{20}$$

$(1 + A^{-1})^{-1} = A/(1 + A) < 1$ is the reduction factor of the evaporation due to the development of a "vapor blanket" under the inversion. The smaller A, i.e. the smaller Q or the larger the denominator of A, the less the evaporation becomes (Case 2).

Case 2

$$Q = 0, \text{ i.e. } A, B = 0.$$

This condition is approximated during periods of cloud cover and might occur also during night.

$$\frac{\partial h}{\partial x}\left[h\left(\Gamma + \frac{L}{c_p}\Gamma_q\right) + \left(\theta_0 - \theta_e + \frac{L}{c_p}(q_0 - q_e)\right)\right] = 0$$

so that $h(x) = h$ becomes:

$$h = -\frac{\theta_0 - \theta_e + (L/c_p)(q_0 - q_e)}{\Gamma + (L/c_p)\,\Gamma_q} > 0.$$

The humidity field along the trajectory under the inversion is given by:

$$\bar{q}(x) = \exp\left(-\frac{c_t}{h}x\right)(\bar{q}_0 - q_0) + q_0; \quad \bar{q}_0 = \bar{q}(x=0).$$

Hence the evaporation becomes negligibly small after long travel distances:

$$E_0(x) = \varrho\, c_t\, u(q_0 - \bar{q}(x)) = \varrho\, c_t u\,(q_0 - \bar{q}_0)\exp\left(-\frac{c_t}{h}x\right),$$

$$\tilde{E}_0(x) = \varrho\, c_t\, u(q_0 - \tilde{\bar{q}}(x)) = \varrho\, c_t\, u(q_0 - \bar{q}_0)\frac{h}{x}\left(1 - \exp\left(-\frac{c_t}{h}x\right)\right).$$

It is clear that the thermal properties may be similarly studied replacing q_0, q_e, Γ_q by θ_0, θ_e, Γ.

Example

As there are only insufficient data available, an example is chosen for the case 1 of a totally mixed environment, where the humidity and temperature

$$q(h) = q_e = q(x=0) \qquad \theta(h) = \theta_e = \theta(x=0)$$

are constant and represent climatological values. The following data from the Lake Mead Studies for June 1953 (Harbeck et al., 1958) are used:

$$\theta_e = 26.5°\mathrm{C}, \quad q_e = 3.7 \text{ g/kg}, \quad u = 5\text{m sec}^{-1}$$

$$\theta_0 = 22°\mathrm{C}, \quad q_0 = 16.1 \text{ g/kg}, \quad Q = 326 \text{ Ly d}^{-1},$$

and $\varrho = 1$ kg/m³; $z_0 = 1$ cm is assumed.

This is directly leads to $c_t = 1.3 \times 10^{-3}$, and $A = 5.0$. The constant slope of the height of the inversion layer (17) $c_t \cdot A = 6.5 \times 10^{-3}$ seems to be within the right order of magnitude. Over Massachusetts Bay (Craig, 1946) the values 15×10^{-3} to 1×10^{-3} are found, decreasing with larger distances from the shore. This is an indication of a parabolic shape (13) of the stable layer for the more general case than discussed here. Furthermore a slope of 5×10^{-3} is obtained by measurements over an irrigated field (Dyer & Crawford, 1965).

The reduction of the evaporation and the turbulent heat flux due to the inversion becomes $A/1 + A = 83.3$ %.

Fig. 1 shows the inversion height. Fig. 2 shows the mean humidity of the inversion layer with the related flux of latent heat. The

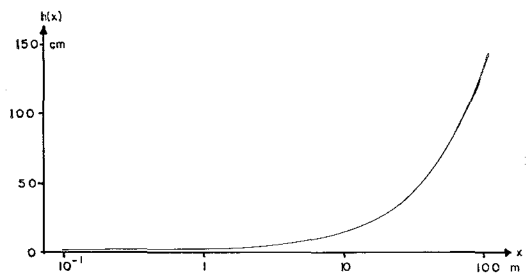

Fig. 1. Inversion height h (x) versus fetch x.

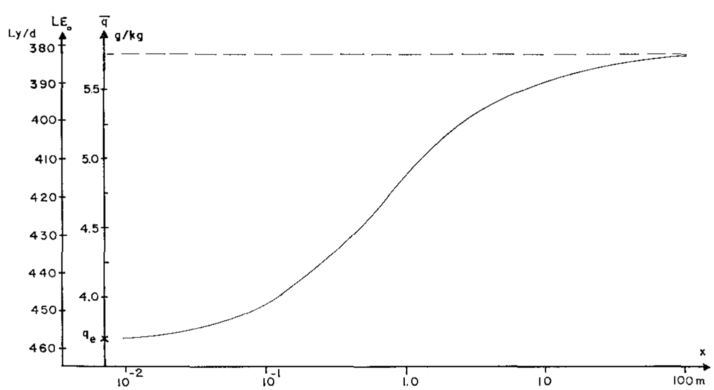

Fig. 2. Mean specific humidity $\bar{q}$ of the inversion layer and the latent heat flux LE_0 depending on the travel distance x of the air over the water surface.

mean temperature of the inversion layer and the heat flux are of course similar. The temperature starts with $\bar{\theta}_0 = \theta_e = 26.5°C$ at the shore and approaches $\bar{\theta} \to 25.8°C$ over the reservoir. The related heat flux starts with $H_0 = -60.7$ Ly d^{-1}.

The evaporation and the turbulent heat flux (20) over the lake approach the values,

$$L\,E_0 \to 383 \text{ Ly d}^{-1} \quad \text{and} \quad H_0 \to -57 \text{ Ly d}^{-1},$$

which compare with the monthly mean data of June 1953 measured at Lake Mead:

$$L\,E_0 = 412 \text{ Ly d}^{-1} \quad \text{and} \quad H_0 = -86 \text{ Ly d}^{-1}.$$

Conclusions

In deserts and semideserts an inversion can reduce the evaporation over lakes and water reservoirs. This will occur preferably during the daytime, when the heated air is advected from the surroundings, supported by evaporational cooling over the water surface.

As a consequence of advection, warm and dry air is introduced to the inversion layer and causes the inversion to rise, whereas substance from beneath cannot penetrate through it. This is the reason, why the evaporation and the turbulent heat flux do not vanish, but approach lower limits already after a relatively short distance off-shore from the land–lake boundary. These limiting values [eq. (20), or for the more general case eq. (15)] are determined by the parameters of the environment and the water surface.

As only the main features are shown, this model neglects all the other dynamical and radiational effects that can get involved in this process, so e.g. the influence of the evaporational cooling on the water surface temperature.

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ОБ ИСПАРЕНИИ ОЗЕРА В ТЕПЛОМ И СУХОМ ОКРУЖЕНИИ

Испарение, подавляемое инверсией, может иметь место над водными резервуарами в случае адвекции воздуха из сухого и теплого окружения. Предлагаемая модель этого процесса основана на сохранении тепла и влаги и на балансе энергии на поверхности воды. В стационарных условиях аналитически определяются высота инверсии и турбулентные потоки тепла и водяного пара. Они зависят от расстояния, проходимого ветром над водной поверхностью, от простых параметров турбулентности, от полного притока радиации к водной поверхности и от распределения температуры и влажности над сушей. Представлен пример в случае хорошо перемешанного окружения.