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Observations and modelling of 1/f-noise in weather and climate

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Abstract. Data with power spectra close to $S(f) \sim 1/f$ is denoted as $1/f$ or flicker noise. High resolution measurements during TOGA/COARE for temperature, humidity, and wind speed (1 min resolution) reveal $1/f$ spectra while precipitation shows no power-law scaling during the same period. However, a binary time series indicating the precipitation events (1 for precipitation, 0 for no precipitation) shows a clear $1/f$ spectrum in line with the remaining boundary layer data. For extreme events in time series with $1/f$ spectra the return time distribution is well approximated by a Weibull-distribution for short and long return times. The daily discharge of the Yangtze river shows high volatility which is linked to the intra-annual $1/f$ spectrum. The discharge fluctuations detected in different time windows are represented by a single function (a so-called data collapse) similar to the universal behavior found for turbulence and various physical systems at criticality. The collapse is well described by the Gumbel distribution.

1 Introduction

Increasing variability at lower frequencies is ubiquitous in meteorology and hydrology (Jiang et al., 2005; Yano et al., 2004; Fraedrich and Blender, 2003). If changes of measurement devices and external impacts can be excluded this volatility is attributed to the internal variability of weather and climate. The low frequency part of the power spectra of these time series can typically be approximated by a power law, $S(f) \sim f^{-\beta}$, with $\beta > 0$, and is denoted as long term memory since the correlation function does not decay in a finite time.

The so-called $1/f$ or flicker noise denotes spectra close to $1/f$. An important property of $1/f$ noise is that the variance of running mean time series does not decay, thus statistical analyses and skillful statistical predictions are hampered. Since exponents in the wide range $\beta = 0.5 \dots 1.5$ are allowed (see for example Voss and Clarke, 1975) these spectra are sometimes called "$1/f$ like". This definition should not be confused with the mathematically sharp definition $\beta = 1$ used in statistics, which is the upper bound for stationarity. In finite observational datasets $1/f$ spectra are limited at low frequencies, hence the process can be considered to be stationary (see Stoisiek and Wolf (1976) for a discussion of stationarity of $1/f$ processes).

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Data with $1/f$ power spectra cover wide ranges of frequencies from minutes to millennia, for example, tropical boundary layer data with 1 min resolution (TOGA/COARE, Yano et al., 2004) and reconstructed temperatures up to the Milankovitch cycle (Huybers and Curry, 2006). Updating the overview by Fraedrich et al. (2009) we present new results on (a) the variability of binary precipitation events in high resolution TOGA/COARE measurements and (b) on the universality of the fluctuations observed in the Yangtze river discharge. Models and the impact of $1/f$ spectra on extreme event distributions are considered briefly.

2 Observing 1/f variability

Examples for $1/f$ spectra are found in wide ranges from minutes to millennia: (i) tropical boundary layer observations (TOGA/COARE, Yano et al., 2004), (ii) the discharge of the Yangtze river in the intra-annual frequency range (Wang et al., 2008), (iii) the sea surface temperature in a region of the North Atlantic and in the southern ocean (reproduced by models, Fraedrich and Blender, 2003), (iv) reconstructed near surface temperatures up to the Milankovitch cycle (Huybers and Curry, 2006).

Tropical rainfall at the TOGA/COARE research vessel Kexue shows no clear power-law scaling in contrast to temperature, humidity, and wind speed which reveal clear $1/f$ spectra. In order to detect a similar behavior in the precipitation time series (Fig. 1a) data transformations are assessed

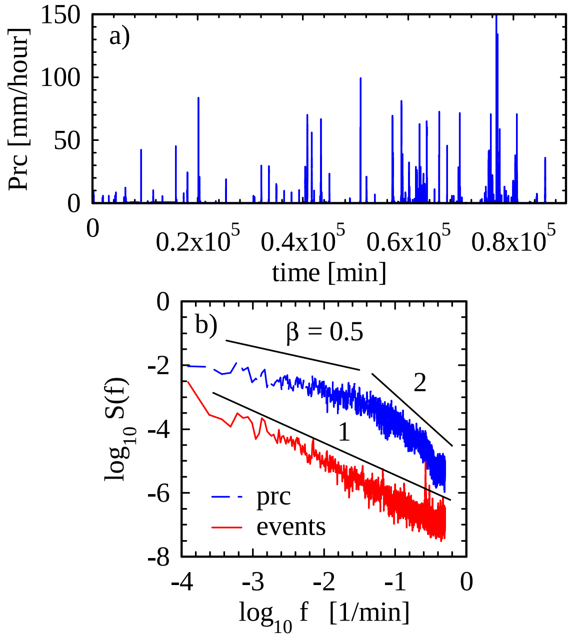

Figure 1. **(a)** Precipitation at the station Kexue (TOGA/COARE), resolution 1 min, **(b)** power spectra (10 window averages) for precipitation and the binary event time series. The slopes are guides for the eye.

providing the following result: precipitation *events*, i.e. the binary sequence defined by 1 for nonvanishing precipitation and 0 otherwise, show a clear $1/f$ spectrum (Fig. 1b). A possible explanation for this feature is that precipitation events are coupled to boundary layer processes (e.g. instability), while the rainfall amount at single stations is determined by other processes.

3 Modeling $1/f$ variability

Whereas red noise is modeled by autoregressive processes (AR), for example with an AR-1 where the coefficient a determines the threshold frequency, there is no simple or generic model for $1/f$ spectra. An increase of the AR-1 coefficient a towards $a \to 1$ increases memory and variance, but does not alter the low frequency plateau of the red noise spectrum. Long term memory and $1/f$ can be mimicked by autoregressive processes in two ways: (i) superposition of AR-1 processes with a continuous band of threshold frequencies and (ii) extension to an infinite number of coefficients defining a fractional AR process (FAR).

A physical model for $1/f$ behavior is given by diffusion in two adjacent compartments with two distinctly different diffusion coefficients (Fraedrich et al., 2004). An upper mixed layer is driven by an uncorrelated atmospheric heat flux and coupled to a deep abyssal ocean with lower diffusivity. This model enables the simulation of the observed spectrum of sea surface temperatures in the North Atlantic and the southern ocean. Based on this model the abyssal diffusivity can be estimated if long term observations of the surface variability are available.

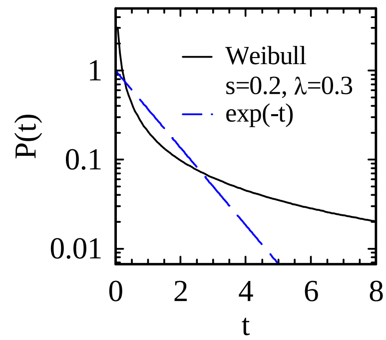

Figure 2. Return time distribution for extremes in $1/f$ (solid, Weibull) compared to the exponential distribution for uncorrelated data (dashed).

4 Extremes and $1/f$ variability

For long term memory processes the statistical properties of extreme event return times are altered. While the return times are exponentially distributed for uncorrelated data, Bunde et al. (2005) suggested a so-called stretched exponential, $\exp(-(t/\lambda)^k)$, for large return times. Blender et al. (2008) observed that the return time distribution for $1/f$ spectra is well described by a Weibull distribution, $f(t) = (k/\lambda)(t/\lambda)^{k-1}\exp(-(t/\lambda)^k)$, which includes a power-law for small return times and an asymptotic stretched exponential for large times (occurring as first and second factors). A main finding is the long term predictability of successive return times.

Employing a surrogate $1/f$ data (simulated by a fractional autoregressive process, FAR) the parameters of the Weibull distribution are estimated (parameters shape $s = 0.2$ and scale $\lambda = 0.3$) leading to a distinctly different behavior of the return times compared to the exponential (Poisson) distribution (Fig. 2). The responsible mechanism is, given a positive threshold, the presence of long term positive (negative) anomalies, which favor long periods of enhanced (reduced) threshold crossings and hence short (long) return times.

5 Universality and $1/f$ variability

The daily Yangtze river discharge measured at the station Yichang (since 1892, located downstream the Three Gorges Project) reveals high intra-annual and inter-annual variability (Fig. 3a). For time scales between two weeks and 1 yr the spectrum follows a $1/f$ power-law (Fig. 3b), which hampers the calculation of a mean annual cycle. Due to this extreme variability the hydrological conditions along the Yangtze

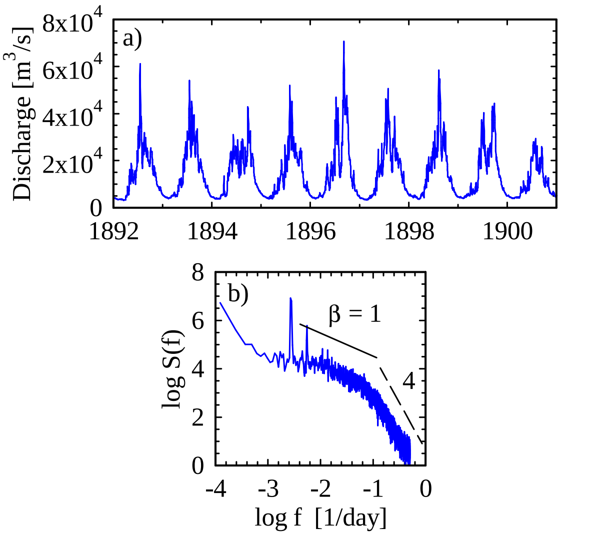

Figure 3. Yangtze river discharge at the station Yichang, **(a)** snapshot of 9 yr, **(b)** power spectrum (1892–2004, 6 segment average), slopes indicate the power-law exponent β.

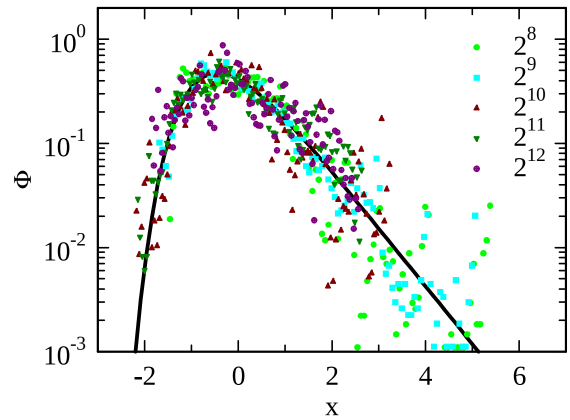

Figure 4. Rescaled frequency distribution of variances in the Yangtze river discharge at Yichang for different segment lengths (in days, see also Fig. 3), the solid line represents the Gumbel distribution (for parameters see text).

river were characterized by extreme floods before the construction of the dam in the year 2006. Precipitation can be excluded as a source for this type of variability since it is less correlated. On interannual time scales the discharge reveals weaker long term memory ($\beta \approx 0.3$, see Wang et al., 2008).

Data with a $1/f$ spectrum are associated with an extreme value distribution (Antal et al., 2001). For a time series $h(t)$, fluctuations $w_T = [h(t) - \overline{h}(t)]^2$ for different segments T (time windows) are determined; μ is the mean and σ the standard deviation of the fluctuations w_T. The distribution $P(w_T)$ of the fluctuations is transformed to a rescaled function $\Phi(x) = \sigma P(w_T)$ depending on the standardized fluctuation $x = (w_T - \mu)/\sigma$. Employing this transformation (Bramwell et al., 1998) leads to a remarkable overlapping of the data (a so-called data collapse) for a large number of physical systems including fluid turbulence. The data for $\Phi(x)$ obtained for the daily Yangtze river discharge versus the standardized fluctuations x collapses for different time segments (Fig. 4). As found by Antal et al. (2001) the result is well represented by the extreme value (Gumbel) distribution $\Phi(x) = a\ \exp(-(ax+\gamma) - e^{-(ax+\gamma)})$, with $a = \pi/\sqrt{6} \approx 1.282$, and the constant $\gamma \approx 0.5772$, capturing the skewness of the non-Gaussian distribution (note that there is no fit applied).

6 Conclusions

Power spectra close to $1/f$ variability are ubiquitous in meteorological and hydrological time series. We present new results obtained for precipitation data with 1 min resolution and for the daily Yangtze river discharge. The frequency range of $1/f$ spectra is limited: at low frequencies either by the duration of the time series or by a regime with less memory, and at high frequencies by the Nyquist frequency or a short term memory regime (for example $S(f) \sim f^{-2}$ or steeper). Since $1/f$ noise is close to nonstationarity, statistical analyses have to be applied carefully because the mean cannot be estimated using averages in finite intervals.

High resolution precipitation data from TOGA/COARE shows no power-law scaling while temperature, humidity and wind speed follow a $1/f$ spectrum for the same measurement period. A binary time series obtained for precipitation events, however, shows a clear $1/f$ power-law scaling. This new finding hints to a coupling of the precipitation events with boundary layer processes, while the rainfall amount is controlled by an independent process.

The exponential distribution of extreme event return times is altered for data with $1/f$ spectra since the low frequency variability can lead to long periods below the threshold (depressions) associated with large return time intervals. Short return times are favored during periods, whose longer term mean is high. We have demonstrated how the Weibull distribution captures both effects.

The daily Yangtze river discharge shows a $1/f$ spectrum on intra-annual time scales. After an adequate rescaling, the discharge fluctuations for different time scales are represented by a single function which is well described by the Gumbel distribution. Thus the fluctuations of the Yangtze river discharge follow a universal distribution known from turbulence and critical phenomena. The physical mechanisms relevant for $1/f$ behavior are still unclear. The ubiquity hints to a general ("universal") or generic property of complex systems, and the absence of characteristic time scales is explained by the scale invariance due to many degrees of freedom. The result for the Yangtze river discharge might be associated with the multitude of time scales involved.

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