

(From the QUARTERLY JOURNAL OF THE ROYAL METEOROLOGICAL SOCIETY, Vol. 98, No. 416, April 1972)

A simple climatological model of the dynamics and energetics of
the nocturnal circulation at Lake Victoria

By K. FRAEDRICH

Department of Atmospheric Science, Colorado State University, Fort Collins, Colorado

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Department of Atmospheric Science, Colorado State University, Fort Collins, Colorado

(Manuscript received 16 July 1971; in revised form 20 October 1971. Communicated by Dr. A. K. Betts)

SUMMARY

The local atmospheric night-time circulation at Lake Victoria (East Africa) is simulated by a simple model which is cylindrically symmetric. It consists of an upper layer (linearized, steady state, hydrostatic, incompressible, saturated over the lake) above a surface boundary layer (constant flux layer, log + lin profile). Analytical solutions are derived for the dynamical, energetical, and hydrological parameters. They mainly depend on the surface temperature difference between land and lake, which is the driving mechanism of the motion, and the choice of representative mesoscale transport coefficients K_m , K_h . The results are compared with the climatological features of the nocturnal circulation.

LIST OF SOME SYMBOLS

- | | | |
|------|-------------------------|--|
| (i) | $h \leq z \leq Z + h$: | upper layer ($1 \geq \xi \geq 0$)
$\gamma_a, \gamma_g, \gamma_m$ temperature gradient: adiabatic, geometric, moist adiabatic,
N Brunt-Vaisälä-frequency,
K_h, K_m turbulent transfer coefficients for heat and momentum,
M_s, q_s mixing ratio and specific humidity of saturation |
| (ii) | $z_0 \leq z \leq h$: | surface boundary layer
L Monin-Obukhov-length,
u_* frictional velocity,
k von Kármán constant,
\bar{p} mean density. |

1. INTRODUCTION

In tropical latitudes a local circulation of thermal and orographic origin has a great influence on the climate and hydrology of its environment. Altogether these diurnal circulations are responsible for a great part of the vertical mass transport of the Hadley cell, although their total area covers less than about 0.5 per cent of the area of the tropical zone (Flohn 1970).

A simple model simulating the climatological features of such a local circulation is proposed for Lake Victoria (0.5° N – 3° S), which shows a marked peak of precipitation; over 500 mm/year more than on the hinterland. The reason for this is a thermal circulation, which is generated almost every night (mainly in the rainy season) by the temperature difference between the land and the lake. As 76 per cent of the water entering Lake Victoria is precipitation on the lake surface (Hutchinson 1957), this local circulation also plays an important role in the hydrology of the lake.

The first part of this paper outlines the dynamics of the model, which is cylindrically symmetric with a circular model lake with a diameter of 300 km (an area equal to that of Lake Victoria). It consists of an upper layer and a flat surface boundary layer underneath. The circulation in the upper layer (i) is described by vertically limited, steady, incompressible, hydrostatic perturbations of a motionless, horizontally homogeneous reference atmosphere, which has a constant local stability. There exists no Coriolis force and the atmosphere over the lake is saturated. The lower boundary conditions are chosen to match

*On leave from Meteorologisches Institut der Universität Bonn.

with the surface boundary layer (ii); the physical conditions at the interface between the two layers are: maximum of radial mass flow, continuity of all velocity components and of the vertical sensible heatflux over the lake. In the surface boundary layer (ii), a constant flux layer, these conditions are fulfilled by the Monin-Obukhov similarity profile, which is radially modified with respect to the upper circulation.

The second part describes the energetics of the model. The energy equation of the upper circulation (i) allows one to calculate the kinetic and available potential energy, the production of kinetic and available potential energy, and the subsequent dissipation or diffusion respectively, as well as the release of latent heat and the condensation rate. These analytical values are compared with empirical data from investigations on tropical disturbances. By including the surface boundary layer (ii) the hydrological cycle of the whole system can be handled and is related to the actual data for Lake Victoria.

2. DYNAMICS

(i) The nocturnal circulation is described by perturbations of the first order, the basic state of which (reference atmosphere: s) is motionless, hydrostatic, and thus horizontally homogeneous:

$$\frac{\partial p_s}{\partial z} = -g\rho_s, \quad p_s = \rho_s R T_s, \quad \theta_s = T_s \left(\frac{p_{sh}}{p_s} \right)^{R/c_p}$$

with a vertically constant local stability:

$$S_s = \frac{\partial \ln \theta_s}{\partial z} = \frac{N^2}{g} = \frac{\gamma_a - \gamma_g}{T_s}.$$

The resulting (tropospheric) temperature distribution

$$T_s = \frac{g}{c_p} \frac{N^2}{g} - \left(\frac{g}{c_p} \frac{N^2}{g} - T_{sh} \right) \exp \frac{N^2}{g} (z - h)$$

fits the tropical standard atmosphere (deviation $< 3^\circ$ at 10 km) for $S_s = N^2/g = 1.5 \cdot 10^{-5} \text{ m}^{-1}$. Reference-pressure p_s and — density ρ_s can also be evaluated. The assumption of a motionless reference atmosphere, necessary to yield an analytically solvable set of equations, is thought acceptable, because the mean tropospheric wind over Lake Victoria is typically 5 m s^{-1} or less (Fraedrich 1968; Fig. 5)

The three-dimensional perturbations (e.g. $A' = A - A_s$, $A'/A_s \ll 1$) in cylindrical co-ordinates are assumed to be hydrostatic, non-divergent, cylindrically symmetric (as the Coriolis force is negligible in equatorial latitudes), and in a steady state. Thus, r , z — co-ordinates are used with the origin in the centre of the lake.

Considering the equation of state:

$$\frac{\rho'}{\rho_s} = \frac{p'}{p_s} - \frac{T'}{T_s} = \frac{c_v}{c_p} \frac{p'}{p_s} - \frac{\theta'}{\theta_s}$$

and defining $\pi = P'/\rho_s$, $\vartheta = \theta'/\theta_s$, the horizontal equation of motion including only the vertical transport of radial momentum, the hydrostatic equation, and the continuity equation are:

$$0 = -\frac{\partial \pi}{\partial r} - K_m \frac{\partial^2 u}{\partial z^2}, \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$0 = -\frac{\partial \pi}{\partial z} + g\vartheta, \quad . \quad . \quad . \quad . \quad . \quad (2)$$

$$0 = \frac{\partial w}{\partial z} + \frac{1}{r} \frac{\partial ru}{\partial r}, \quad . \quad . \quad . \quad . \quad . \quad (3)$$

An equation for the tangential velocity does not appear owing to the cylindrical sym-

metry and negligible Coriolis force. In Eq. (2) the multiplier S_s has been neglected compared to the operator $\partial/\partial z$.

The first law of thermodynamics contains two terms: the vertical divergence of turbulent transfer of heat, and the (irreversible) release of latent heat by condensation of water vapour only over the lake, the radius of which is $r_i = 150$ km:

$$0 = -Sw + K_h \frac{\partial^2 \vartheta}{\partial z^2}; \quad K_h, K_m \text{ constant}, \quad (4)$$

where $S = \begin{cases} -l^2 S_s: 0 < r < r_i & \text{over the lake} \\ S_s: r_i < r < r_a & \text{over the land, } r_a: \text{lateral boundary,} \end{cases}$

because $-l^2 S_s = \frac{\partial \ln \theta_s}{\partial z} - \frac{L}{c_p T_s} \frac{\partial m_s}{\partial z} = \frac{\gamma_m - \gamma_g}{T_s}$

and $l^2 = \frac{\gamma_g - \gamma_m}{\gamma_a - \gamma_g} = \text{constant.}$

Non-dimensionalizing the co-ordinates by the vertical extension Z of the circulation and reducing the system (1) to (4) to one partial differential equation, a dimensionless number, a Rayleigh-number:

$$R = \frac{S_s \cdot g}{K_h \cdot K_m} Z^4$$

can be introduced:

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial \vartheta}{\partial r} \right) = - \frac{\partial^6 \vartheta}{\partial z^6} R^{-1} \begin{cases} -l^2: 0 \leq r \leq r_i \\ 1: r_i \leq r \leq r_a. \end{cases} \quad (5)$$

Separation of the dependent variable ($\vartheta = \vartheta(r) \cdot \vartheta(z)$) gives two Bessel equations for the horizontal distribution:

$$\frac{1}{r} \frac{d}{dr} \left(r \frac{d\vartheta(r)}{dr} \right) = \vartheta(r) \cdot \lambda^2 \cdot R^{-1} \begin{cases} -l^2 \text{ (index } i): 0 \leq r \leq r_i \\ 1 \text{ (index } a): r_i \leq r \leq r_a, \end{cases} \quad (6)$$

where λ^2 is a parameter of separation.

There are five boundary conditions:

at $r = 0$: $u_i = 0$; $\vartheta_i = 1$, normalized by the amplitude $\vartheta_h = \Delta \theta / \theta_{sh}$, where $\Delta \theta$ is the temperature difference between the lake centre and the outmost limit of the circulation at $z = h$, and θ_{sh} the temperature of the reference atmosphere at $z = h$;

at $r = r_a$: either ϑ_a or $u_a = 0$;

at $r = r_i$: $u_i(r_i) = u_a(r_i)$ kinematic and

$\pi_i(r_i) = \pi_a(r_i)$ dynamic condition at the vertical interface.

If the lateral boundary r_a moves to infinity the radial (and dimensionless) solutions, shown in Fig. 1, are easily deduced (Kuo 1965; Lilly 1960; Haque 1952) leaving the dimensional constants in the vertical solution:

	$0 \leq r \leq r_i$	$r_i \leq r < \infty$	}	
$\vartheta(r) [= B(\vartheta)] = -\pi(r) =$	$F_0 \left(\frac{\lambda}{l\sqrt{R}} r \right)$	$-\frac{1}{l} \frac{F_1(a)}{H_1^1(ila)} \cdot i \cdot H_0^1 \left(i \frac{\lambda}{\sqrt{R}} r \right)$		(6a)
$u(r) \left[= \frac{\sqrt{R}}{\lambda l} B(u) \right] =$	$\frac{\sqrt{R}}{\lambda l} \cdot F_1 \left(\frac{\lambda}{l\sqrt{R}} r \right)$	$\frac{\sqrt{R}}{\lambda l} \cdot \frac{F_1(a)}{H_1^1(ila)} \cdot H_1^1 \left(i \frac{\lambda}{\sqrt{R}} r \right)$		
$w(r) =$	$\frac{1}{l^2} F_0 \left(\frac{\lambda}{l\sqrt{R}} r \right)$	$-\frac{1}{l} \frac{F_1(a)}{H_1^1(ila)} \cdot i \cdot H_0^1 \left(i \frac{\lambda}{\sqrt{R}} r \right)$		

where $i = \sqrt{-1}$; $F_{0,1}, H_{0,1}$ are Bessel, Hankel functions (Jahnke, Emde and Lössch 1960). The normalized functions $B(\vartheta), B(u)$ for the radial profile of ϑ and u in the range $0 \leq r < \infty$ will be used later to describe the horizontal structure of the surface boundary layer (Eqs. (8), (11) and (13)). A good approximation for the parameter of separation becomes $\lambda = l \cdot a \cdot \sqrt{R}/r_i$ with $a = \pi/4 + \arctan l$.

ϑ, u, π are continuous at r_i . A discontinuity in w occurs at the land-lake vertical interface. This will be a strong gradient in nature.

An equation similar to the shallow convection type (Chandrasekhar 1961) yields the vertical distribution for the given λ :

$$\frac{d^6 \vartheta(z)}{dz^6} = -\lambda^2 \vartheta(z). \quad (7)$$

There are three boundary conditions at the top

$$\frac{\partial u}{\partial z} = w = \vartheta = 0 \quad \text{or} \quad \vartheta = \frac{\partial^2 \vartheta}{\partial z^2} = \frac{\partial^4 \vartheta}{\partial z^4} = 0,$$

and three at the bottom (a no-slip condition; a vertical velocity, and a temperature amplitude depending on the interaction with the surface boundary layer):

$$\frac{\partial u}{\partial z} = 0, \quad w = w_h \frac{K}{S_s Z^2}, \quad \vartheta = \vartheta_h = \frac{\Delta \theta}{\theta_{sh}}.$$

With these boundary conditions the appropriate vertical solution is:

$$\left. \begin{aligned} \vartheta(\xi) &= \vartheta_h \frac{1}{3} \sum_{j=1}^3 \frac{\text{sh } q_j \xi}{\text{sh } q_j} - \frac{w_h}{\lambda^2} \frac{1}{3} \sum_{j=1}^3 q_j^4 \frac{\text{sh } q_j \xi}{\text{sh } q_j}, \\ u(\xi) &= -\frac{K_h}{S_s Z^2} \left\{ \vartheta_h \frac{1}{3} \sum_{j=1}^3 q_j^3 \frac{\text{ch } q_j \xi}{\text{sh } q_j} + w_h \frac{1}{3} \sum_{j=1}^3 q_j \frac{\text{ch } q_j \xi}{\text{sh } q_j} \right\}, \\ w(\xi) &= -\frac{K_h}{S_s Z^2} \left\{ \vartheta_h \frac{1}{3} \sum_{j=1}^3 q_j^2 \frac{\text{sh } q_j \xi}{\text{sh } q_j} + w_h \frac{1}{3} \sum_{j=1}^3 \frac{\text{sh } q_j \xi}{\text{sh } q_j} \right\}, \\ \pi(\xi) &= -gZ \left\{ \vartheta_h \frac{1}{3} \sum_{j=1}^3 q_j^5 \frac{\text{ch } q_j \xi}{\text{sh } q_j} + w_h \frac{1}{3} \sum_{j=1}^3 q_j^3 \frac{\text{ch } q_j \xi}{\text{sh } q_j} \right\}, \end{aligned} \right\} \quad (7a)$$

where $\xi = 1 - \left(z - \frac{h}{Z}\right)$; $q_j = \lambda^{\frac{1}{2}} \left(i, \frac{1}{2}(\sqrt{3} + i), \frac{1}{2}(\sqrt{3} - i)\right)$; $\lambda \neq \pm n\pi^{\frac{2}{3}}$;

sh, ch are hyperbolic functions.

The product of Eqs. (6a) and (7a) is a particular solution of Eqs. (1) to (4) and describes the part of the circulation which lies above the surface boundary layer.

(ii) For simplicity some approximations must be made in the surface boundary layer. The radial velocity $u(r)$ in the boundary layer will be assumed to have the same structure $B(u)$ as in the upper circulation, although a vertical log-linear profile will be used (strictly valid only for horizontal homogeneity). A continuous vertical velocity at the interface between the layers is thus possible.

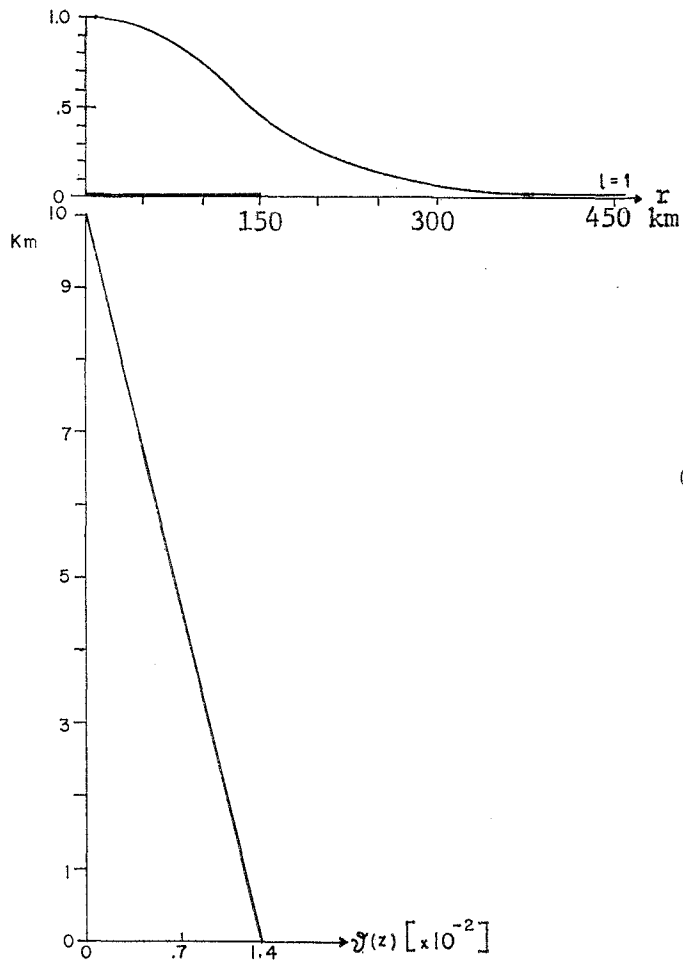
The equations for the boundary layer are:

$$u = \frac{u_*}{k} \left(\ln \frac{z}{z_0} + \frac{\beta}{L} (z - z_0) \right) B(u); \quad (8a)$$

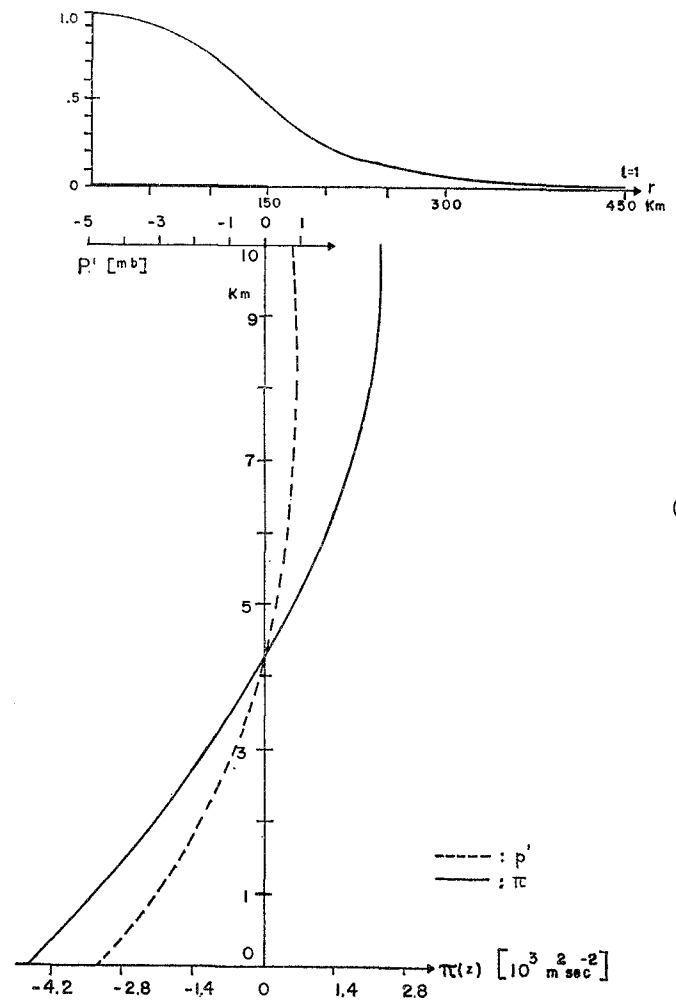
$$\frac{\partial u}{\partial z} = \frac{u_*}{k} \left(\frac{1}{z} + \frac{\beta}{L} \right) B(u); \quad (8b)$$

$$w = - \int_{z_0}^z \frac{1}{r} \frac{\partial}{\partial r} (ru) dz'; \quad (9)$$

$$\theta = - \frac{H}{c_p k \bar{p} u_*} \left(\ln \frac{z}{z_0} + \frac{\beta}{L} (z - z_0) \right) + \theta_0(r); \quad (10)$$

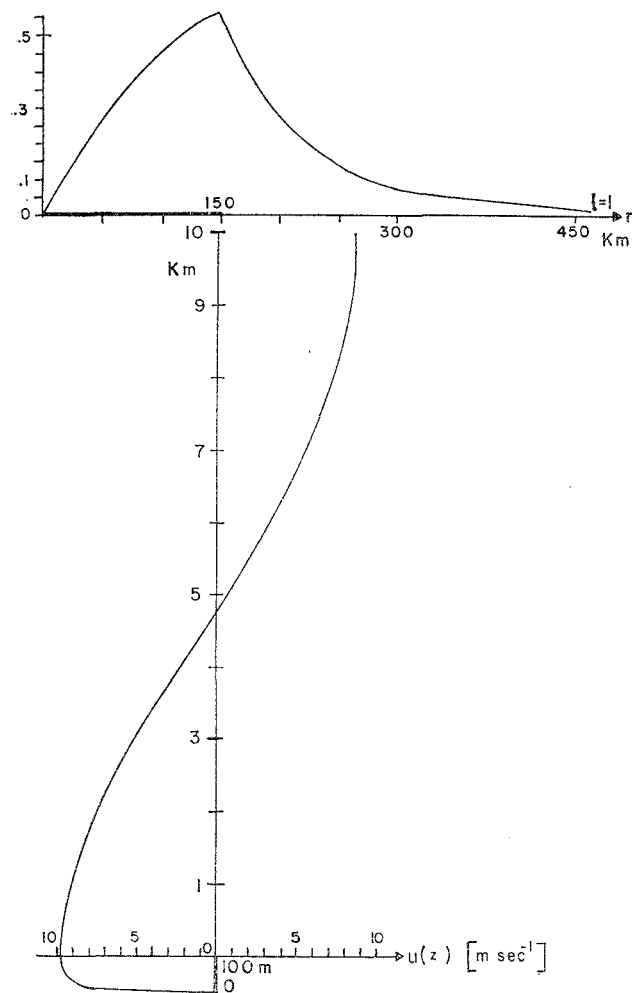


(a)

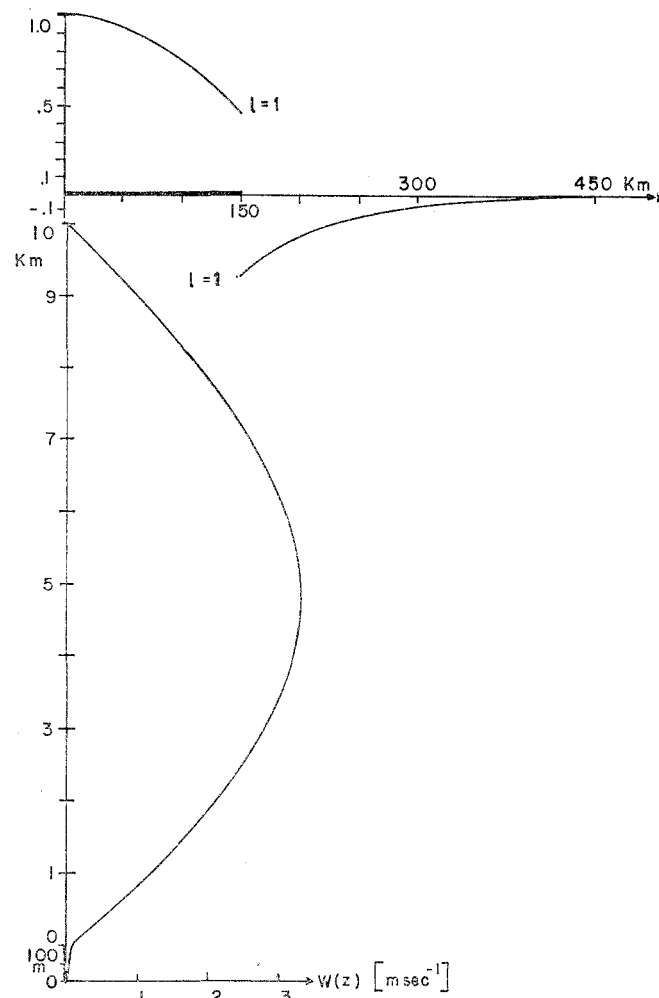


(b)

Figure 1. Caption on facing page.



(c)



(d)

Figure 1. (a) Temperature and (b) pressure perturbation, (c) radial and (d) vertical velocity; top section of each Figure: radial variation (dimensionless): $\vartheta(r)$, $\pi(r)$, $u(r)$, $w(r)$; bottom section of each Figure: vertical variation: $\vartheta(z)$, $\pi(z)$, $u(z)$, $w(z)$.

These must fulfill the following physical conditions at the horizontal interface between both layers.

(a) A maximum of convergence caused by surface friction occurs in the lower parts of synoptic scale disturbances. Here, with respect to the area mean of the horizontal divergence: $\overline{\text{div}_h \mathbf{u}} = 2u/r$, the necessary conditions for such a vertical extremum is:

$$\left. \frac{\partial u}{\partial z} \right|_h = 0.$$

This is assumed to exist at the height h , where the surface boundary layer ends (and the upper circulation starts, with $\partial u/\partial z = 0$ (no-slip) at the bottom). The no-slip condition at the bottom of the upper circulation implies no vertical flux of momentum from below. This is of no dynamical consequence, since the magnitude of the stress ($\bar{\rho} u_*^2$) in the surface boundary layer (constant flux layer) is negligible compared to representative values for the momentum transport in the upper layer.

In Eq. (8b) $\partial u/\partial z = 0$ at $z = h$ implies

$$\frac{1}{h} = -\frac{\beta}{L}.$$

This condition will be used to eliminate L from Eqs. (8a) and (10). Although L has some radial dependence as the heat flux changes over the lake, this will be neglected using a constant value of h .

This leads directly to the velocity field at $z = h$, ($h \gg z_o$):

$$\left. \begin{aligned} u(h, r) &\doteq \frac{u_*}{k} \ln h/ez_o B(u); & e &\doteq 2.71 \dots \\ w(h, r) &\doteq \frac{u_* h}{k Z} \ln h/e^{3/2} z_o \frac{1}{r} \frac{\partial}{\partial r} (rB(u)). \end{aligned} \right\} \quad (11)$$

(b) The continuity of the radial velocity between both layers results in an expression for the frictional velocity (using Eqs. (7a) and (11))

$$u_* \doteq \frac{k}{\ln h/ez_o} \frac{\sqrt{R}}{\lambda l} \frac{K_h}{S_s z^2} \frac{\Delta \theta}{\theta_{sh}} \frac{1}{3} \sum_{j=1}^3 - q_j^3 \frac{\text{ch } q_j}{\text{sh } q_j}. \quad (12)$$

This gives the vertical velocity in Eq. (11) as a boundary value for the upper circulation (w_h) and thus fulfills the kinematic interface condition. The resulting coefficient of turbulent diffusion for momentum (and heat, if the Prandtl-number $\text{Pr} = 1$)

$$K_{m,h} = u_* k \frac{z}{h} h/(h - z)$$

will reach the value for the upper circulation only a negligibly small distance under the top of the boundary layer.

(c) To close the system of equations and to calculate the amplitudes of the circulation, the temperature difference $\Delta \theta = \vartheta_h - \theta_{sh}$ at $z = h$ between lake centre and outermost lateral boundary must be related to the sensible heat flux at the surface z_o of the lake (also the flux at the top of the surface boundary layer, the bottom of the upper circulation).

This sensible heat flux will be taken as

$$H = H_o \cdot B^2(\vartheta). \quad (13)$$

With this form, continuity of sensible heat flux leads to the simple condition at the top of the constant flux layer:

$$H_o = c_p \bar{\rho} \cdot w(h, r = 0) \cdot \Delta \theta, \quad (14)$$

H_o : sensible heat flux at the centre of the lake.

This is valid over the lake, since the radial profiles of H and $w \cdot \vartheta \cdot \theta_{sh}$ have been chosen

the same. Using Eqs. (11), (12) and (14)

$$\Delta\theta^2 = \frac{H_o}{c_p \bar{\rho} h} \frac{S_s Z^3}{K_h} \theta_{sh} \frac{\ln h/ez_o}{\ln h/e^{3/2}z_o} \left(\frac{1}{3} \sum_{j=1}^3 - q_j^3 \frac{ch q_j}{sh q_j} \right)^{-1} \quad (15)$$

The heat flux profile actually depends on complex relations between the upper atmospheric circulation and the surface z_o^* , but these are well approximated by Eq. (13).

The vertical temperature difference between the surface z_o and the top h of the boundary layer (Eq. (10)):

$$\begin{aligned} -\delta\theta = \theta(h, r) - \theta_o(r) &= -\frac{H_o B^2(\vartheta)}{c_p k \bar{\rho} u_*} \cdot \ln h/ez_o \\ &= -\frac{h}{k^2 Z} \frac{\lambda}{\sqrt{R}} \cdot \ln h/ez_o \cdot \ln h/e^{3/2}z_o \cdot \Delta\theta \cdot B^2(\vartheta) \end{aligned} \quad (16)$$

shows that the stratification $-\delta\theta/h$ stabilizes from an unstable gradient above the lake to the adiabatic lapse rate as one approaches the lateral boundary. $(\Delta\theta + \delta\theta)$ at $r = 0$ is the surface temperature difference between the lake centre and the distant undisturbed environment, which must be prescribed to determine the intensity of the whole circulation.

(iii) The whole circulation within both layers can be calculated using Eqs. (8), (9), (10), (12), (15) and (16) for the surface boundary layer, and Eqs. (6a) and (7a) for the upper circulation.

The following data were chosen for a complete calculation of this circulation (Fraedrich 1971):

$z_o \leq z \leq h$: $z_o = 1$ cm, $h = 100$ m, $\Delta\theta + \delta\theta = 5.5^\circ$ C, $\bar{\rho} = 1.06$ g m $^{-3}$, $\theta_{sh} = 295^\circ$ K; $h \leq z \leq Z + h$: $Z = 10$ km, $r_t = 15$ (dimensionless), $K_h = K_m = 5 \cdot 10^6$ m 2 s $^{-1}$, $l^2 = 1$.

The above magnitude of the coefficients K_m , K_h has been deduced from the time and horizontal length scale ($t_s \sim 4$ h, $L_s \sim 300$ km) through the relationship $K = L_s^2/t_s$ (see Diffusion Diagram, Lettau 1951, Fig. 1). These values represent the exchange process due to mesoscale phenomena. They essentially determine the magnitude of the meteorological parameters.

Some results (e.g. Fig. 1) are as follows:

At the horizontal interface between the layers the temperature perturbation at the centre of the lake is $\vartheta_h = 1.4 \times 10^{-2} = 4^\circ/295^\circ$. The sensible heat flux through the constant flux layer at the lake centre becomes $H_o = 85$ ly day $^{-1}$, and the frictional velocity is $u_* = 0.49$ m s $^{-1}$. The mean convergence above the lake yields $\text{div}_h u = -7.5 \times 10^{-5}$ s $^{-1}$ at the top of the surface boundary layer. This is comparable with the convergence maximum of $-3 \cdot 10^{-5}$ s $^{-1}$ between 100 and 200 m determined from pilot balloon data around Lake Victoria (Fraedrich 1968).

3. ENERGETICS

(i) The energy equation of the circulation above the boundary layer results as the sum of the horizontal equation of motion (Eq. (1)), multiplied with $u\rho_s$, and the first law of thermodynamics (Eq. (4)), multiplied with $\vartheta\rho_s \cdot g/S_s$. Both equations are assumed to be time dependent, i.e., to contain the local time derivatives of u , ϑ respectively (Van Mieghem 1963).

$$\frac{\partial}{\partial t} \left(\frac{u^2}{2} + \frac{\vartheta^2 g}{2 S_s} \right) \rho_s - \left(u K_m \frac{\partial^2 u}{\partial z^2} + \vartheta K_h \frac{\partial^2 \vartheta}{\partial z^2} \right) \rho_s = -\rho_s u \frac{\partial \pi}{\partial r} - \rho_s w \vartheta g \quad \begin{cases} -l^2: 0 \leq r \leq r_t \\ 1: r_t \leq r < \infty. \end{cases}$$

*At the Earth surface the energy flux budget $Q = H + E + S$ and the net radiation at night $Q = C_1 - E_1$ lead to the flux of sensible heat $H = (C_1 - E_1) - (E + S)$; H depends on the net longwave radiation $C_1 - E_1$ (C_1 : longwave counter-radiation; E_1 : longwave emission) and the sum of the latent heat flux E and the heat flux into the ground S . Leaving $E + S$ constant but different over the lake and land, the horizontal variation of H is determined by the net radiation. On account of the decreasing cloudiness from the centre of the lake outward, Q also decreases. If the discontinuities of $E + S$ and $C_1 - E_1$ at the border land/lake are the same, H is continuous at this boundary. So the continuous quadratic horizontal temperature profile (developed by the upper circulation) proves suitable for the horizontal profile of the sensible heat flux from the surface.

The definition of the available potential energy of perturbations of the reference atmosphere allows a clearer interpretation of the upper energy equation

$$\frac{\partial}{\partial t}(K + A) + D(K) + D(A) = -P(K) - P(A) \quad . \quad . \quad (17)$$

in terms of the volume integrals of the

$$\begin{aligned} \text{kinetic energy} \quad K &= \int_v \rho_s \frac{u^2}{2} dv, \\ \text{its production} \quad P(K) &= \int_v \rho_s u \frac{\partial \pi}{\partial r} dv, \\ \text{its dissipation} \quad D(K) &= - \int_v \rho_s u K_m \frac{\partial^2 u}{\partial z^2} dv, \\ \text{available potential energy} \quad A &= \int_v \rho_s \frac{g}{S_s} \frac{\vartheta^2}{2} dv, \\ \text{its production} \quad P(A) &= \int_v \rho_s l^2 gw \vartheta dv, \\ \text{its diffusion} \quad D(A) &= - \int_v \rho_s \vartheta K_h \frac{\partial^2 \vartheta}{\partial z^2} dv. \end{aligned}$$

K and A are constant, as the circulation is steady ($\partial/\partial t = 0$). The release of latent heat of condensing water vapour $P(L)$ can be derived from the first law (Eq. (4)):

$$\begin{aligned} 0 &= l^2 S_s w + K \frac{\partial^2 \vartheta}{\partial z^2}, \\ - \frac{L}{c_p T_s} w \frac{\partial m_s}{\partial z} &= S_s w - K \frac{\partial^2 \vartheta}{\partial z^2}. \end{aligned}$$

Addition and integration lead to

$$\begin{aligned} P(L) &= - \int_v \rho_s L w \frac{\partial m_s}{\partial z} dv \\ &= \int_v c_p T_s \rho_s (1 + l^2) S_s w dv. \quad . \quad . \quad . \quad (18) \end{aligned}$$

Using the solutions (6a) and (7a), and the density ρ_s and temperature T_s , which vary only in the vertical, the results of the volume integration, first vertically (per unit area: m^{-2}) and then over the whole area, are shown in Table 1.

TABLE 1. ENERGY VALUES OF THE MODEL OF THE LOCAL NOCTURNAL CIRCULATION OVER LAKE VICTORIA

	per unit area ($\times \text{m}^{-2}$)	0 — 150 km	≥ 150 km	Total
K	1.41×10^6	1.95×10^{16}	1.49×10^{15}	$3.44 \times 10^{15} \text{J}$
A	1.77×10^5	3.7×10^{16}	1.88×10^{15}	$5.58 \times 10^{15} \text{J}$
$P(K), -D(K)$	96.6	1.33×10^{13}	1.02×10^{13}	$2.35 \times 10^{13} \text{W}$
$P(A), -D(A)$	96.6	2.02×10^{13}	1.02×10^{13}	$3.04 \times 10^{13} \text{W}$
$P(L)$	9.9×10^8	5.05×10^{14}	—	$5.05 \times 10^{14} \text{W}$
$e = P(K)/P(L)$	—	0.26%	—	0.47%

These analytically calculated values of the nocturnal circulation of Lake Victoria can be compared with the empirical data of e.g.

- (a) A tropical disturbance (in the Gulf of Mexico: 15–18 September 1957; Riehl 1959);
 - (b) Hurricane *Hilda* (October 1964; Anthes and Johnson 1968; Hawkins and Rubsam 1968);
 - (c) Mean tropical cyclones (Palmén and Riehl 1957);
- (Table 2).

TABLE 2. ENERGY DATA OF TROPICAL DISTURBANCES

K	Disturbance (Gulf of Mexico): (Riehl 1959)	$3.0\text{--}8.0 \times 10^{17}\text{J}$
$P(K)$	Disturbance (Gulf of Mexico): (Riehl 1959)	$4.2\text{--}19.0 \times 10^{12}\text{W}$
	<i>Hilda</i> (Hawkins and Rubsam 1968): 0 – 150 km:	$8.6 \times 10^{12}\text{W}$
	Mean tropical cyclones, total: (Palmén and Riehl 1957)	$15.0 \times 10^{12}\text{W}$
$D(K)$	Disturbance (Gulf of Mexico): (Riehl 1959)	$1.8\text{--}9.0 \times 10^{12}\text{W}$
	<i>Hilda</i> (Hawkins and Rubsam 1968): 0 – 150 km:	$2.0 \times 10^{12}\text{W}$
$P(A)$	<i>Hilda</i> (Anthes and Johnson 1968):	$10.3 \times 10^{12}\text{W}$
$P(L)$	Disturbance (Gulf of Mexico): (Riehl 1959)	$2.3\text{--}7.0 \times 10^{14}\text{W}$
$e = P(K)/P(L)$	Disturbance (Gulf of Mexico): (Riehl 1959)	0.8–4.1%

The kinetic energy at Lake Victoria is very small but the latent heat release is nearly as much as in the disturbance over the Gulf of Mexico. The production of kinetic and available potential energy calculated by the model of the local night-time circulation over Lake Victoria amounts to about 30 per cent of tropical cyclones. Thus, the effectivity $e = P(K)/P(L)$ of the local system (normally in tropical cyclones: $e = 1 - 2$ per cent) becomes less and reaches values of immature tropical disturbances. These disturbances exist several days, the completely developed local circulation lasts only a few hours, but it occurs nearly every night especially during the rainy season.

The release of latent heat $P(L)$ is defined as the area integral of the condensation rate C :

$$\begin{aligned}
 C(r) &= \int_0^1 \frac{c_p}{L} T_s \rho_s (1 + l^2) S_s w(\xi) d\xi F_o \left(a \frac{r}{r_i} \right) \\
 &= 14.2 \text{ mm h}^{-1} F_o \left(a \frac{r}{r_i} \right), \quad (19)
 \end{aligned}$$

where $1 \text{ cm h}^{-1} \triangleq 1 \text{ g cm}^{-2} \text{ h}^{-1}$.

So the intensity of condensation C over the model lake becomes

at its coast ($r = r_i$): $C_c = 6.7 \text{ mm/hr}$

at its centre ($r = 0$): $C_M = 14.2 \text{ mm/hr}$

averaged over its circular area A_l : $\bar{C} = 10.2 \text{ mm/hr}$,

where

$$\bar{C} = \frac{1}{A_l} \int_{A_l} C(r) da = \frac{1}{\pi r_i^2} \int_0^{r_i} 2\pi r C(r) dr.$$

The intensity of precipitation P , however, is merely a fraction $a = P/C$ of the condensation rate* and becomes comparable to observations with $a \geq 8$ per cent.

at the coast: $P_c = 0.53$ mm/hr

at the centre: $P_M = 1.14$ mm/hr

averaged: $\bar{P} = 0.81$ mm/hr.

Measurements of raindrop spectra (Diem 1965) in Entebbe at the coast of Lake Victoria show two maxima of rain intensity depending on the spectrum width. The narrow spectrum (< 1.75 mm) with rain intensities < 0.6 mm/hr was assumed to be due to the influence of Lake Victoria; the maximum $1.2 - 3.0$ mm/hr was associated with overlying synoptic disturbances (Fraedrich 1968). The annual average of the maximum rain intensity of < 1.0 mm/hr appears during the nocturnal circulation between 5 to 9 hr, i.e. on 170 rainy days about 10 per cent of the annual total 1,585 mm will rain during each of these 4 hr. (Extremum: 1.35 mm/hr at 4 hr — 5 hr in May (long rains)). The rain over the hinterland of Lake Victoria can be estimated at 1,000 mm/yr. At the coast (Entebbe) it rains $r_c = 585$ mm/yr more and the island Bufumira represents the rain maximum in the middle of Lake Victoria which is $r_m = 1,290$ mm/yr higher (E.A.M.D.). This increase of observed rain intensities agrees with the calculated condensation or precipitation intensities:

$$\frac{C_M}{C_c} = \frac{P_M}{P_c} = \frac{r_m}{r_c} = 2.2.$$

Roughly estimated the nocturnal circulation exists 1,000 hr per year (in order to produce 585 mm/yr by an intensity of 0.53 mm/hr) or 4 hr per day if the system develops 250 times; i.e. the circulation runs stationary for 4 hr ($t = \max |\overline{\text{div}_n u}|^{-1} = 3.7$ hr). This is confirmed by the number of thunderstorm days: 242 per year at Kampala (WMO 1956), and by the diurnal variation of rain at all stations in the vicinity of the lake; e.g. Entebbe: the maxima of rain appear in May 3 hr — 9 hr; December to February 4 hr — 7 hr; June to August 7 hr — 9 hr (East African Meteorological Department 1965).

(ii) Further information can be evaluated from the surface boundary layer:

(a) The dissipation of kinetic energy ϵ :

$$\begin{aligned} \epsilon &= \int_{v_s} \bar{\rho} u_*^2 \frac{\partial u}{\partial z} dv & v_s &= A \cdot h: \text{ volume of the surface} \\ & & & \text{boundary layer} \\ &= \bar{\rho} \frac{u_*^3}{k} \left(\ln \frac{h}{z_0} - 1 \right) \cdot \int_A B(u) da. & da &= 2\pi r dr; \quad A = \pi r^2. \end{aligned}$$

over the lake 0–150 km: 4.00×10^{10} W

over the land > 150 km: 1.57×10^{10} W

total: 5.57×10^{10} W

(Hurricane *Hilda* 0–150 km: 4.20×10^{12} W (Hawkins and Rubsam 1968)).

(b) The production of sensible heat $P(H)$ by the surface boundary layer:

$$\begin{aligned} P(H) &= \int_A H da & H &= H_o \cdot B^2(\vartheta): \text{ sensible heat flux,} \\ & & H_o &= 85 \text{ ly day}^{-1} = 41.5 \text{ W m}^{-2}, \end{aligned}$$

over the lake: 18.5×10^{11} W

over the land: 2.4×10^{11} W

total: 20.9×10^{11} W

The mean flux of sensible heat \bar{H} over the lake into the upper layer:

$$\bar{H} = \frac{1}{A_l} \int_{A_l} H da = 26.3 \text{ W m}^{-2}.$$

*There is only a large heat release, if the droplets do not evaporate but fall as rain. Smaller rain- than condensation-intensity without evaporation of the droplets affecting the system is possible, if the droplets that do not rain are carried into the high troposphere.

(c) The vertical water vapour transport W over the lake through the upper limit h of the surface boundary layer where saturation exists:

$$W = w q_s \bar{p}$$

at the coast ($r = 150$ km): $W = 0.33$ mm/hr

at the centre ($r = 0$): $W = 0.78$ mm/hr

averaged: $W = 0.55$ mm/hr

(d) The mean evaporation \bar{E} over the lake:

$$\bar{E} = \bar{W} - K,$$

i.e. the vertical water vapour transport \bar{W} reduced by the convergence of advected water vapour in the surface boundary layer over the lake K :

$$\begin{aligned} K &= \bar{p} \bar{q} \frac{2}{r_i} \int_0^h u(r_i) dz \\ &= 0.38 \text{ mm/hr,} \end{aligned}$$

where $\bar{q} = 75$ per cent $\times q_s(r_i) = 1.4 \times 10^{-2}$.

So the mean evaporation becomes:

$$\bar{E} = \bar{W} - K = 0.17 \text{ mm/hr.}$$

Using the mean flux of sensible heat \bar{H} , a mean Bowen-ratio \bar{B} over the lake during the night-time circulation can be calculated:

$$\bar{B} = \bar{H}/\bar{L}\bar{E} = 0.22,$$

which is comparable to the Bowen-ratio in the trade wind régime.

(e) The mean net gain of precipitation NP over the lake, produced by the local circulation:

$$NP = \bar{P} - (\bar{W} - K)$$

$$= 0.64 \text{ mm/hr}$$

$$= 640 \text{ mm/yr.}$$

or

as this model night-time circulation lasts 4 hr during 250 days or 1,000 hours per year.

An average run-off RO from the lake is assumed to be

$$RO = 1400 \text{ tons s}^{-1} \text{ (Moerth 1964)}$$

or

$$= 6.45 \text{ mm yr}^{-1}.$$

(Area of the lake $A_l = 68500$ km².)

Both run-off and net precipitation gain by the local circulation are of the same magnitude. One might conclude that the water supply from this circulation over the lake feeds the outflow, or even that the nocturnal circulation over Lake Victoria is the real source of the River Nile.

4. CONCLUSIONS

This simple model analytically solvable shows reasonably good agreement with the local atmospheric circulation at Lake Victoria. The results do depend to some extent on the particular choice of values taken for K_m , K_h and the agreement with observations actually obtained is to some extent an indication that the values chosen are the appropriate ones to be used in this particular model. The climatological features and even the hydrological cycle (assuming a proportional relation of the precipitation rate to the condensation rate) are reproduced. This model was developed to complete a series of various investi-

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gations on the nature of the local atmospheric circulation at Lake Victoria and its meteorological and climatological consequences (Flohn and Fraedrich 1966; Fraedrich 1968). Similar effects have been described at Lake Titicaca (Kessler and Monheim 1968).

In numerical models on the general circulation of the Tropics, these local and geographically well defined systems should be incorporated. A great part of the vertical mass transport within the Hadley cell is due to such stationary, topographically induced circulations. The production of available potential energy presents itself as a quantity for the parameterization of these (perturbation) systems.

ACKNOWLEDGMENTS

Thanks are due to Professor H. Flohn and Professor E. Lingelbach, Universität Bonn, Meteorologisches Institut, for numerous stimulating discussions, to Dr. A. K. Betts, Colorado State University, Fort Collins, Department of Atmospheric Science for carefully reading the manuscript.

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